Algorithms for Hard Problems on Low Highway Dimension Graphs

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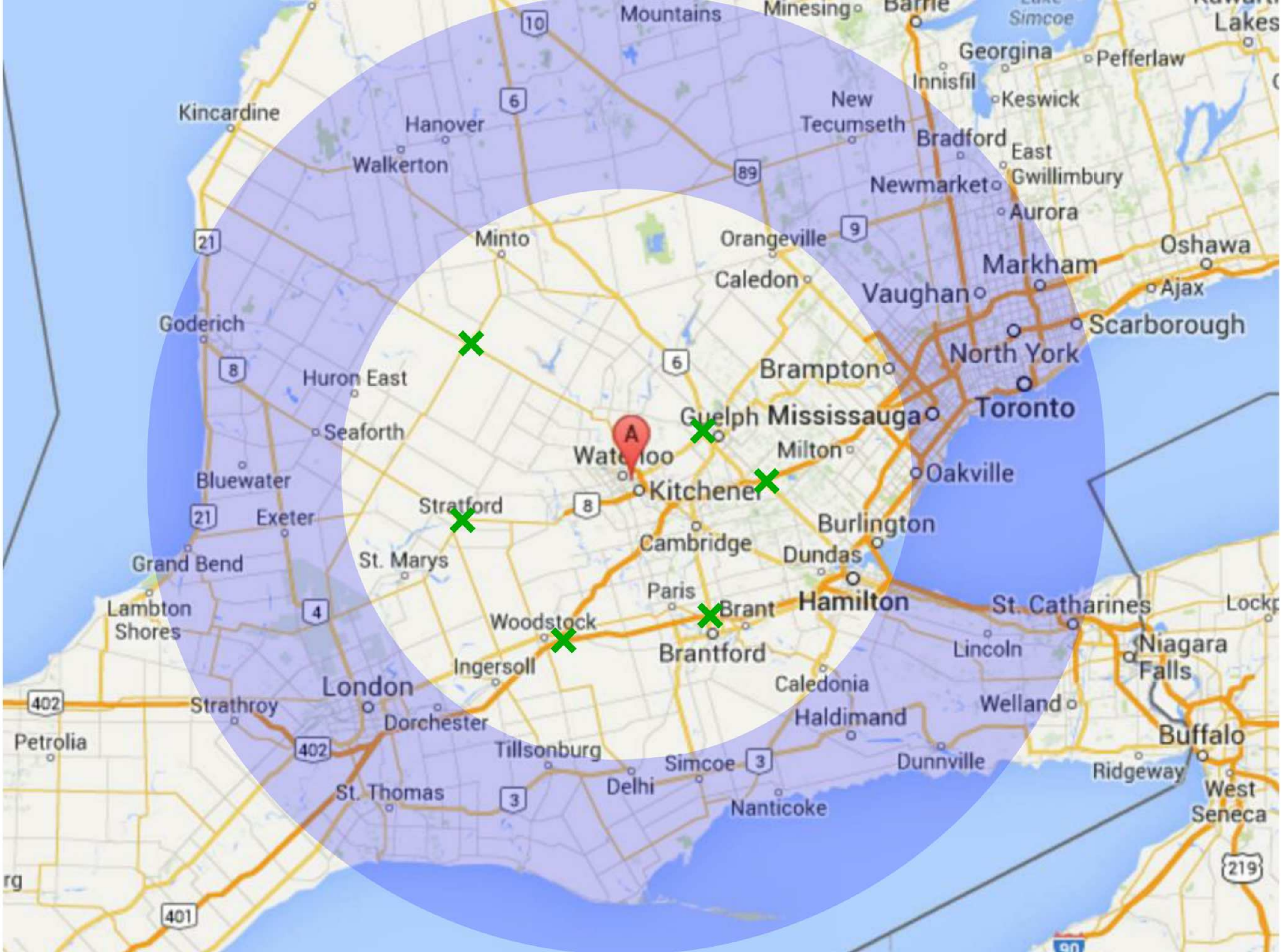

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Aurora
Minto
Orangeville
Oshawa
Markham
Caledon
Vaughan
Ajax
Goderich
Scarborough
North York
Brampton
Huron East
Toronto
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Oakville
Bluewater
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Stratford
Exeter
Burlington
Cambridge
Grand Bend
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Dundas
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Brant
St. Catharines
Woodstock
Brantford
Niagara Falls
Lincoln
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Caledonia
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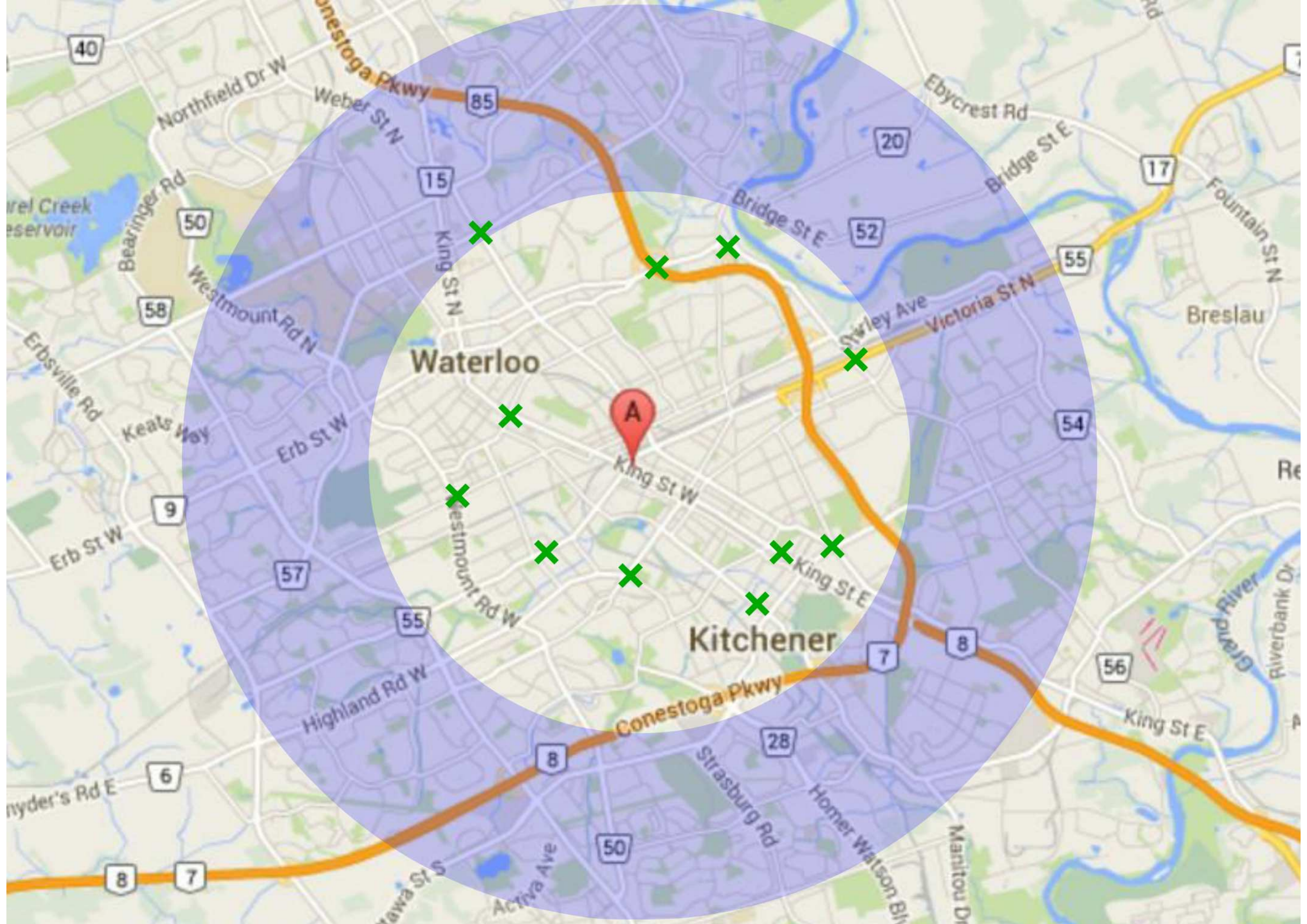
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Erb St W
Highland Rd W
Northfield Dr W
Weber St N
Bridge St E
Ebycrest Rd
Fountain St N
Victoria St N
Bearinger Rd
Erbsville Rd
Keats Way
Strasburg Rd
Homer Watson Bl
Manitou Dr
Activa Ave
Snyder's Rd E
Riverbank Dr
Grand River
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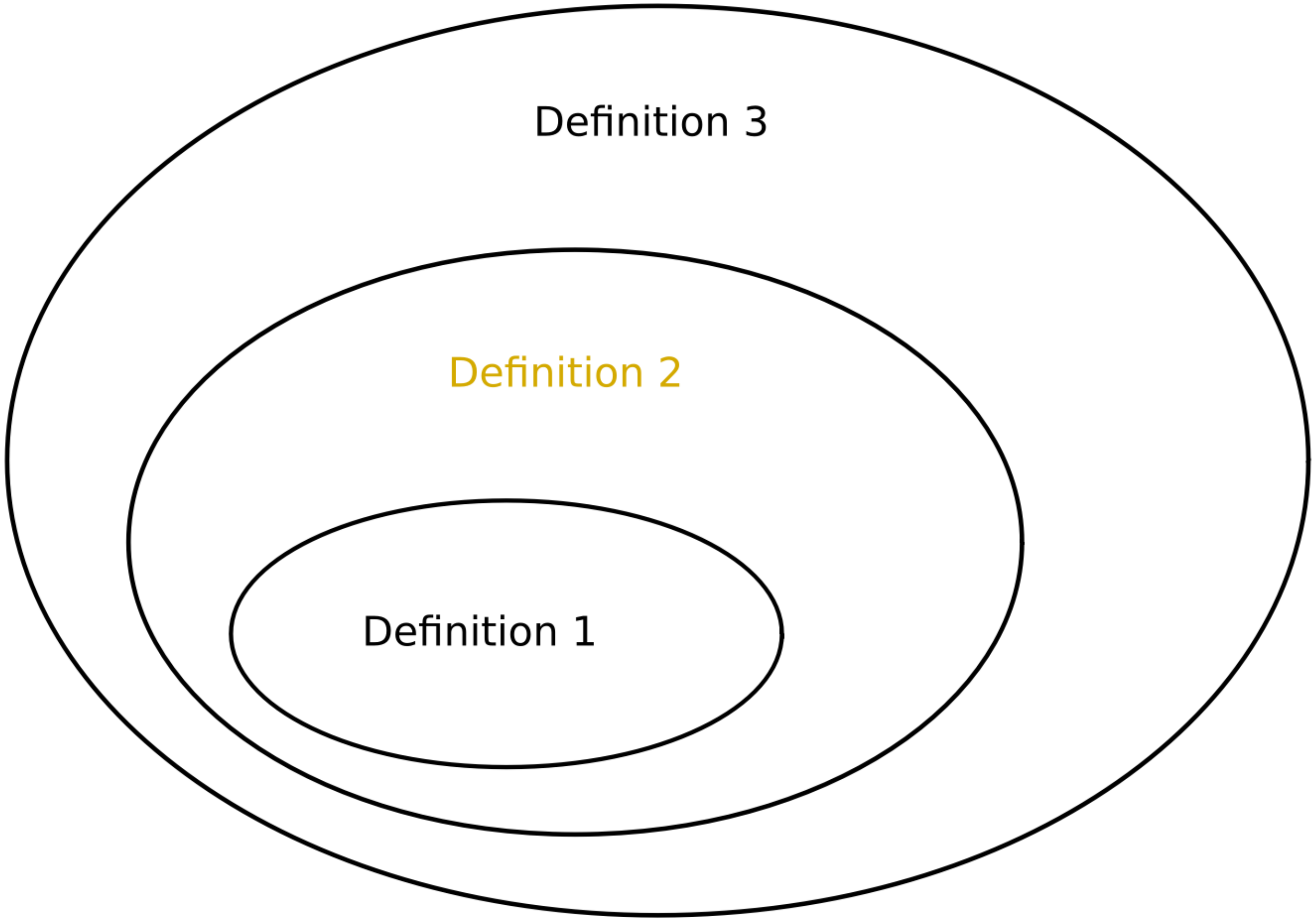
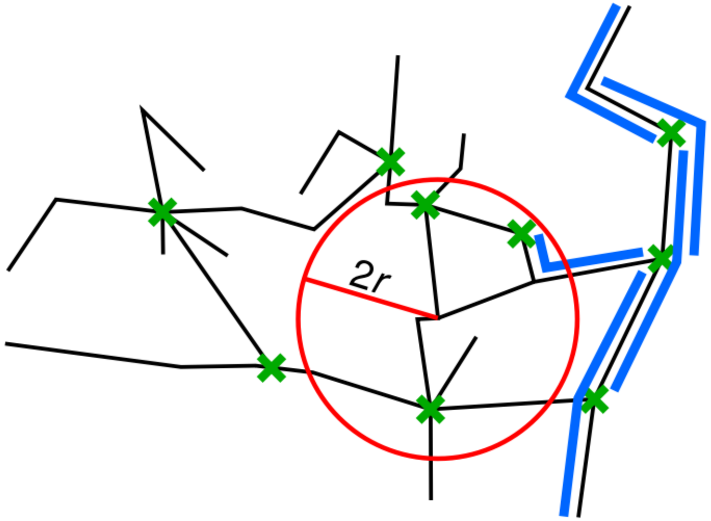
The Highway Dimension

[Abraham et al. '10, '11, '13]



Shortest-Path Covers

[Abraham et al. '10, '11]



Consider:

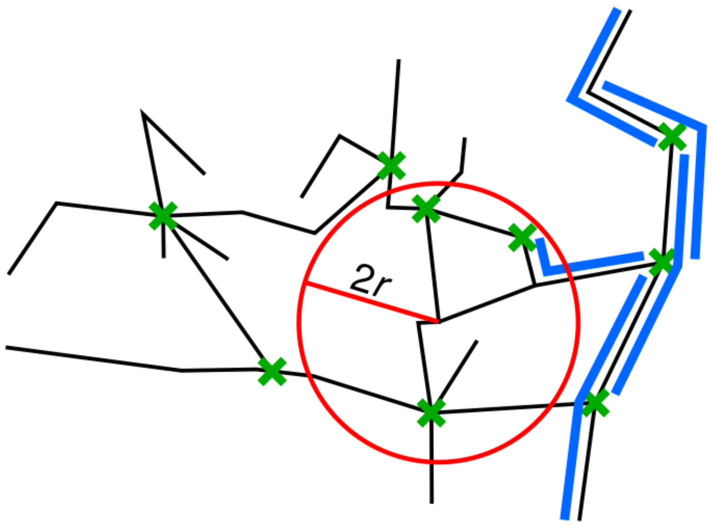
- scale $r \in \mathbb{R}^+$,
- set system $\mathcal{P}_{(r,2r]}$ of all paths that are
  (i) shortest for some $u, v \in V$ and (ii) of length in $(r, 2r]$.

Shortest-path cover:

- hitting set $\text{SPC}(r) \subseteq V$ for $\mathcal{P}_{(r,2r]}$
  $\rightarrow$ hubs
- for every ball $B_{2r}(v) := \{v \in V \mid \text{dist}(v, u) \leq 2r\}$:
  $|B_{2r}(v) \cap \text{SPC}(r)| \leq h \rightarrow$ locally h-sparse

Shortest-Path Covers

[Abraham et al. '10, '11]



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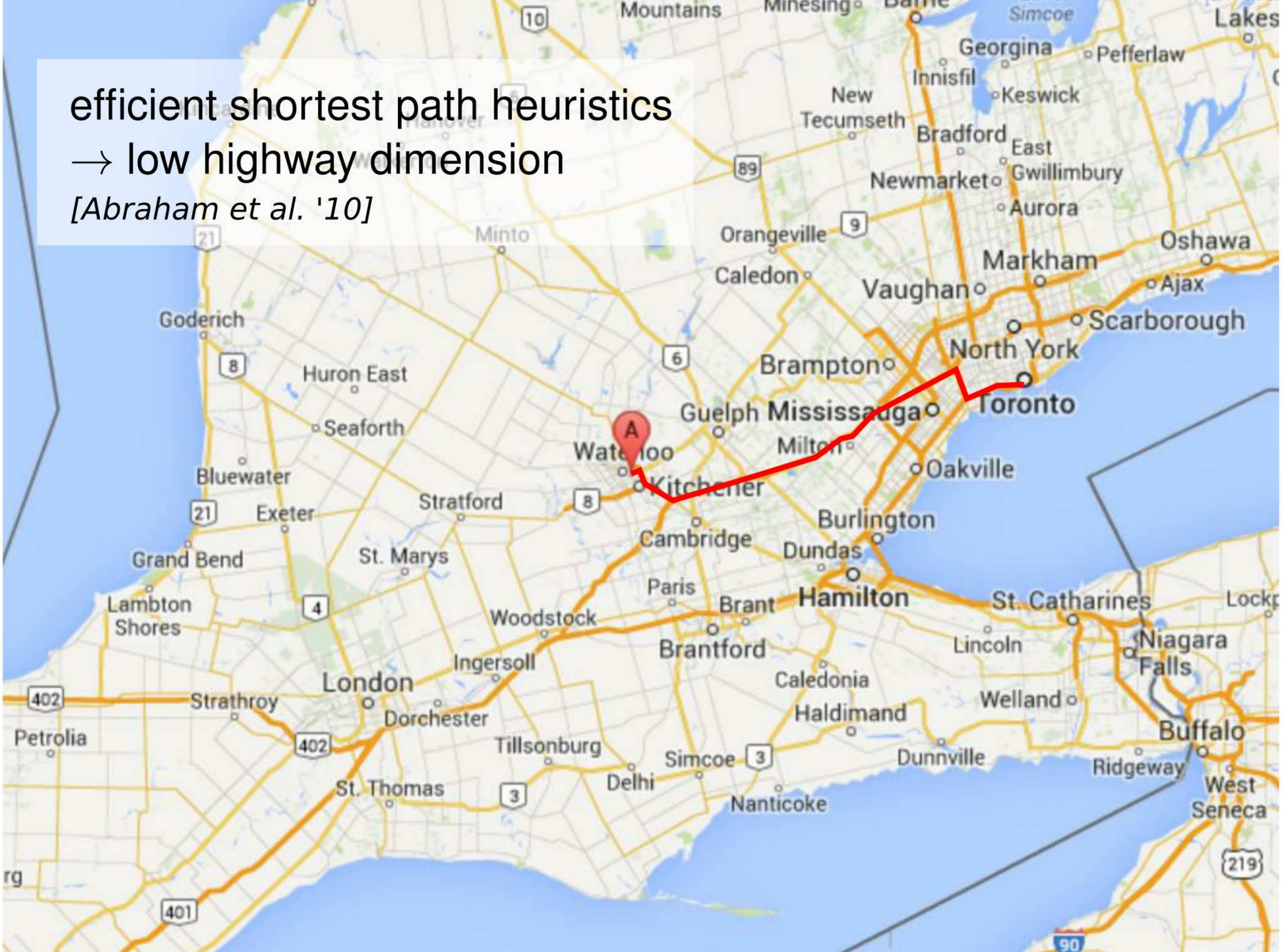
Highway dimension $h \Rightarrow$

- there is locally h-sparse $\textsc{spc}(r)$ for every scale r,
- locally $O(h \log h)$-sparse shortest-path covers
  can be computed (NP-hard in general).

efficient shortest path heuristics

$\rightarrow$ low highway dimension

[Abraham et al. '10]



Travelling Salesperson (TSP):

– given:
  - graph $G = (V, E)$
  - edge weights $w : E \to \mathbb{R}^+$

– find:
  - closed walk (tour) T visiting all of V
  - min $w(T)$

NP-hard problems

Facility Location:

– given:

- graph $G = (V, E)$
- edge weights $w : E \to \mathbb{R}^+$
- opening costs $c : V \to \mathbb{R}^+$

– find:

- facilities $F \subseteq V$
- min $\sum_{v \in V} \text{dist}(v, F) + \sum_{u \in F} c(u)$

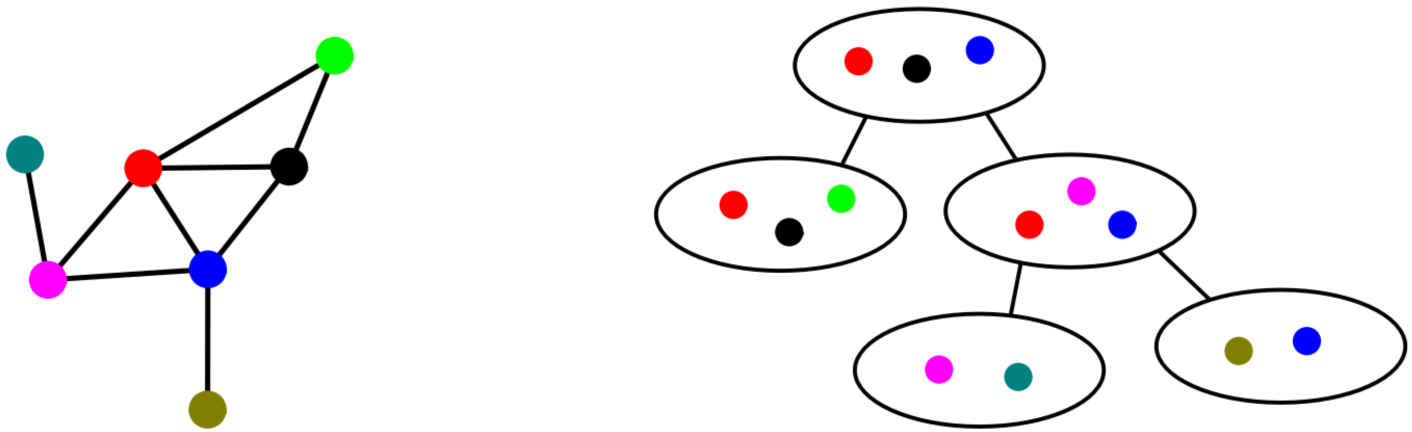
NP-hard problems

Treewith

Tree decomposition of graph $G = (V, E)$:
tree T and bags $X_w \subseteq V$ for each $w \in V(T)$ s.t.

1. $\bigcup_{w \in V(T)} X_w = V$
2. $\forall uv \in E, \exists w \in V(T)$: $u, v \in X_w$
3. $\forall v \in V$: $T[\{w \in V(T) \mid v \in X_w\}]$ is connected

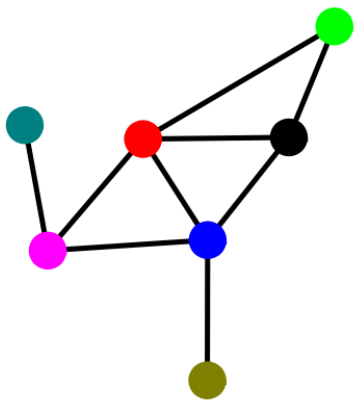
Width: $\max\{|X_w| - 1 \mid w \in V(T)\}$
Treewidth of G: min width of any tree decomposition

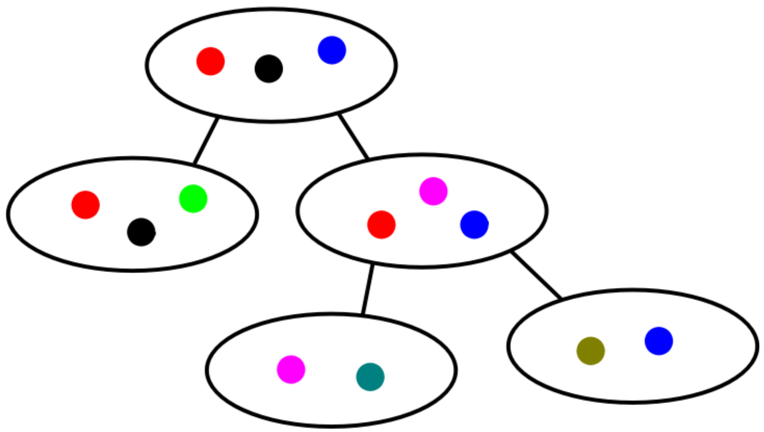


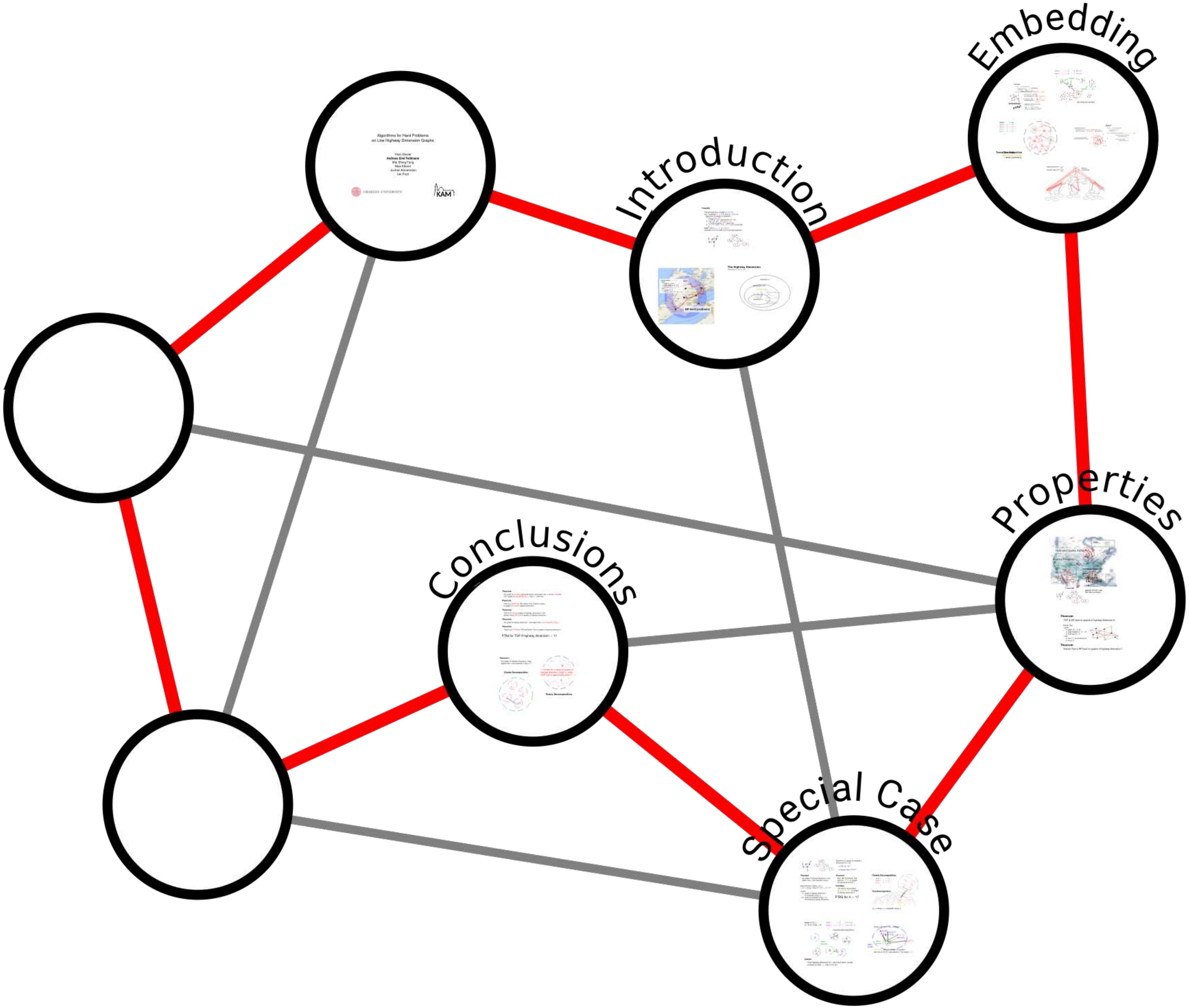
Treewith

Algorithms for graphs of treewidth t:

- TSP: $2^{O(t)} n^{O(1)}$ *[Bodlaender et al. '03]*
- Facility Location: $n^{O(t)}$ *[Ageev '92]*
- ...







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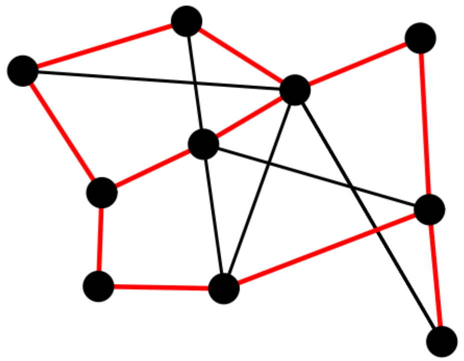
Graph H with $V = V(H) = V(G)$ s.t.



Embeddings

1. small distortion of distances:

   $\forall u, v \in V : \text{dist}_G(v, u) \leq \text{dist}_H(v, u) \leq \delta \, \text{dist}_G(v, u).$

   stretch/distortion

2. structurally simple:

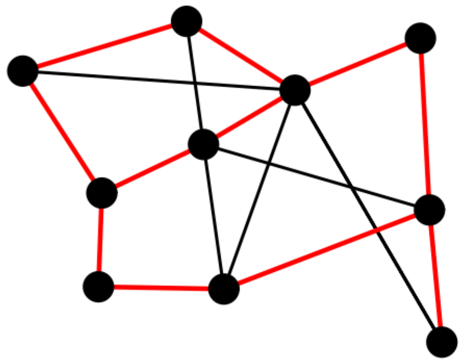
   bounded treewidth

Theorem

For any $\varepsilon > 0$ and any graph with

- highway dimension h
- aspect ratio $\alpha = \frac{\text{max dist}}{\text{min dist}}$

there is a polynomial-time computable probabilistic embedding with



1. small distortion of distances:

   expected distortion $1 + \varepsilon$

2. structurally simple:

   treewidth $(\log(\alpha))^{O(\log^2(h/\varepsilon))}$

Embeddings

Theorem

For any $\varepsilon > 0$ and any graph with

- highway dimension h
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1. small distortion of distances:

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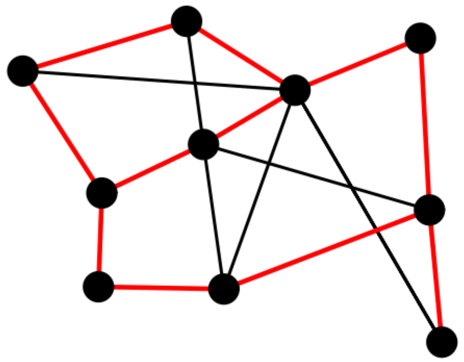
2. structurally simple:

   treewidth $(\log(\alpha))^{O(\log^2(h/\varepsilon))} \leq n^{O(1)}$ for constant h, ε



Embeddings

Theorem

For any $\varepsilon > 0$ and any graph with

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- aspect ratio $\alpha = \frac{\text{max dist}}{\text{min dist}} \leq 2^n$ n/ε using standard trick (with small modification)

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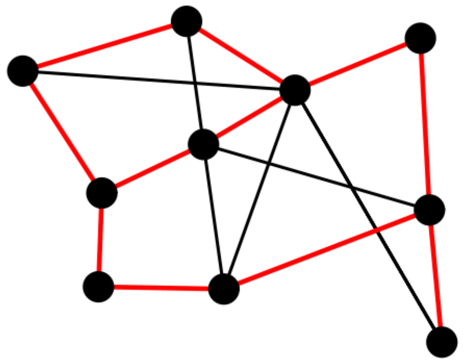
   polylog(n/ε)



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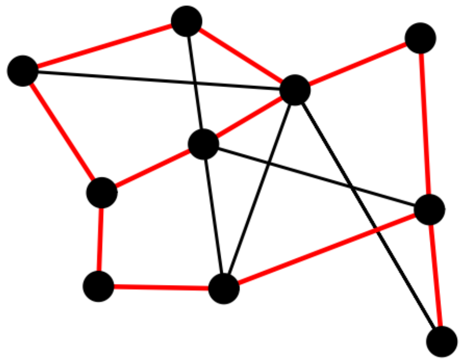

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Embeddings

$\Rightarrow$ QPTAS for TSP, Facility Location, ...
(APX-hard in general)

PTAS $\rightarrow$ $(1+\varepsilon)$-approx. in $n^{O(1)}$ time
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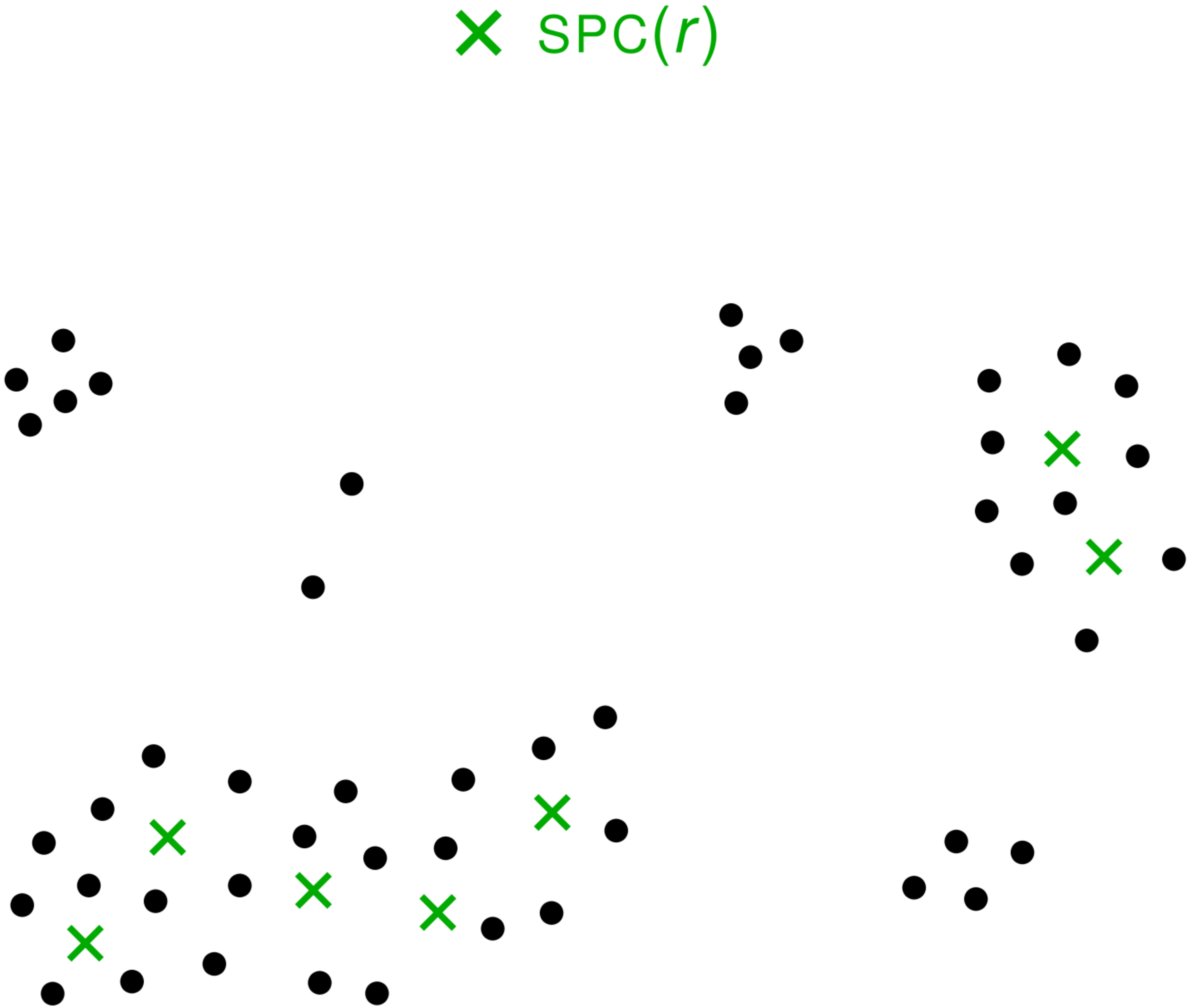
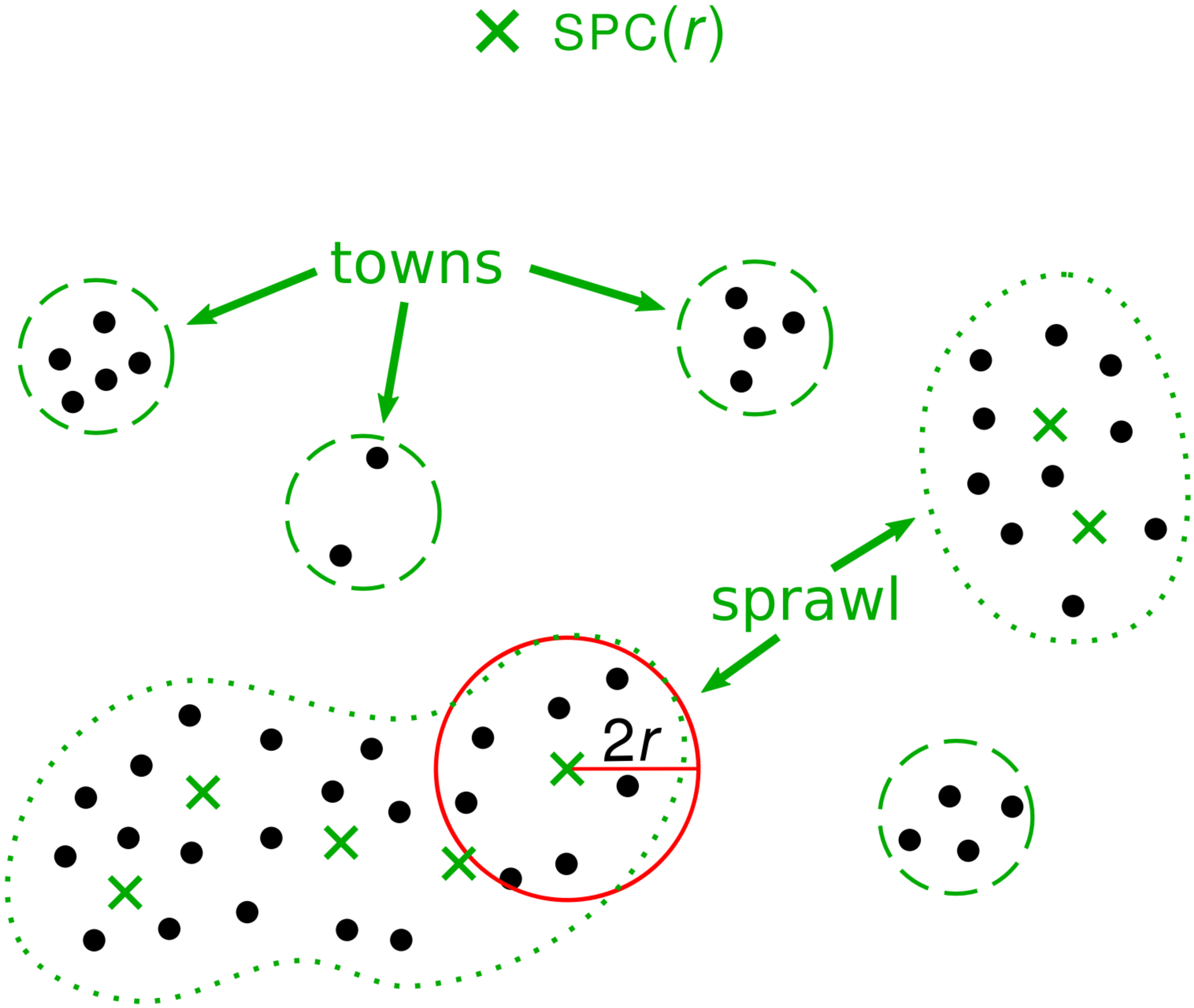
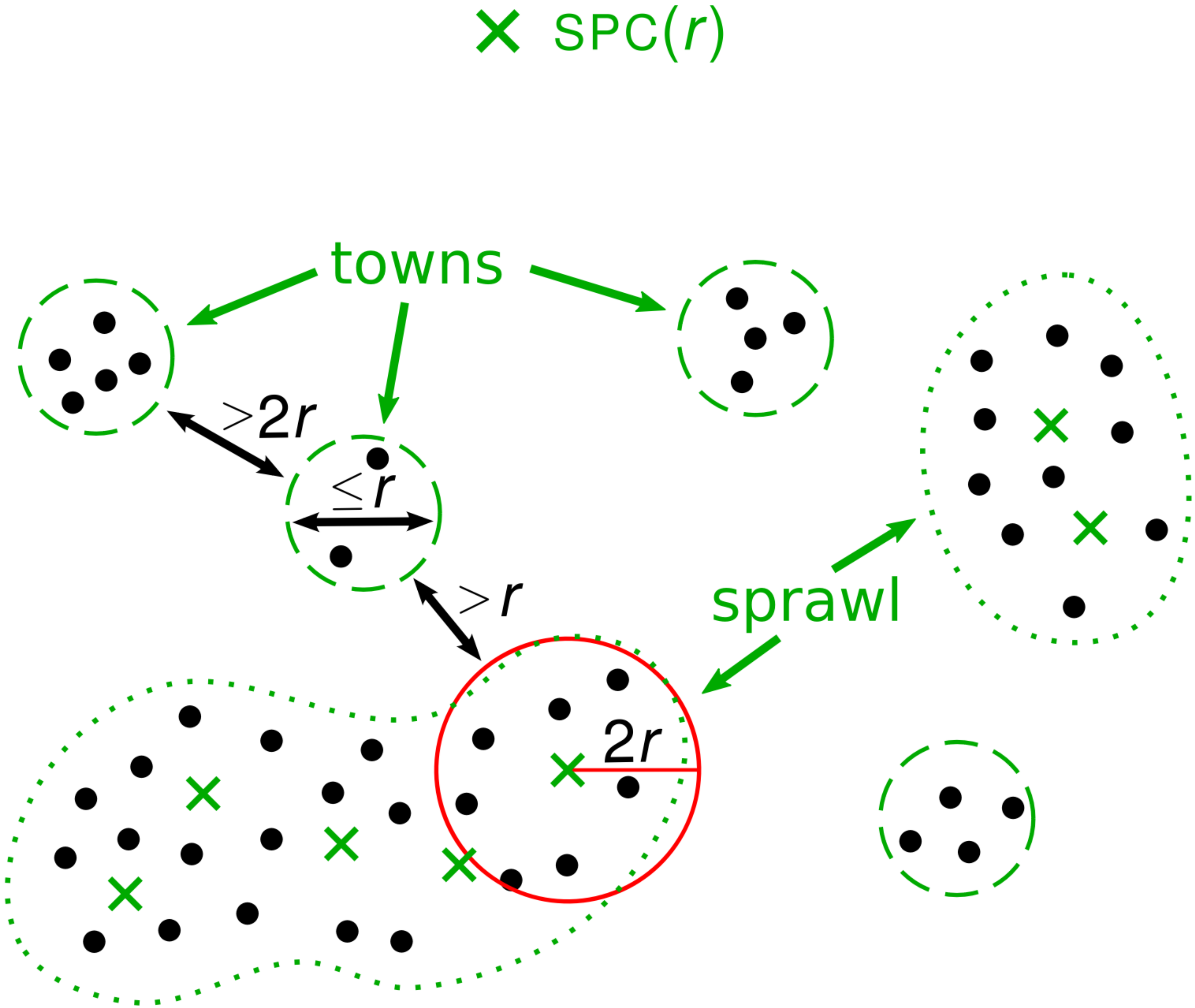
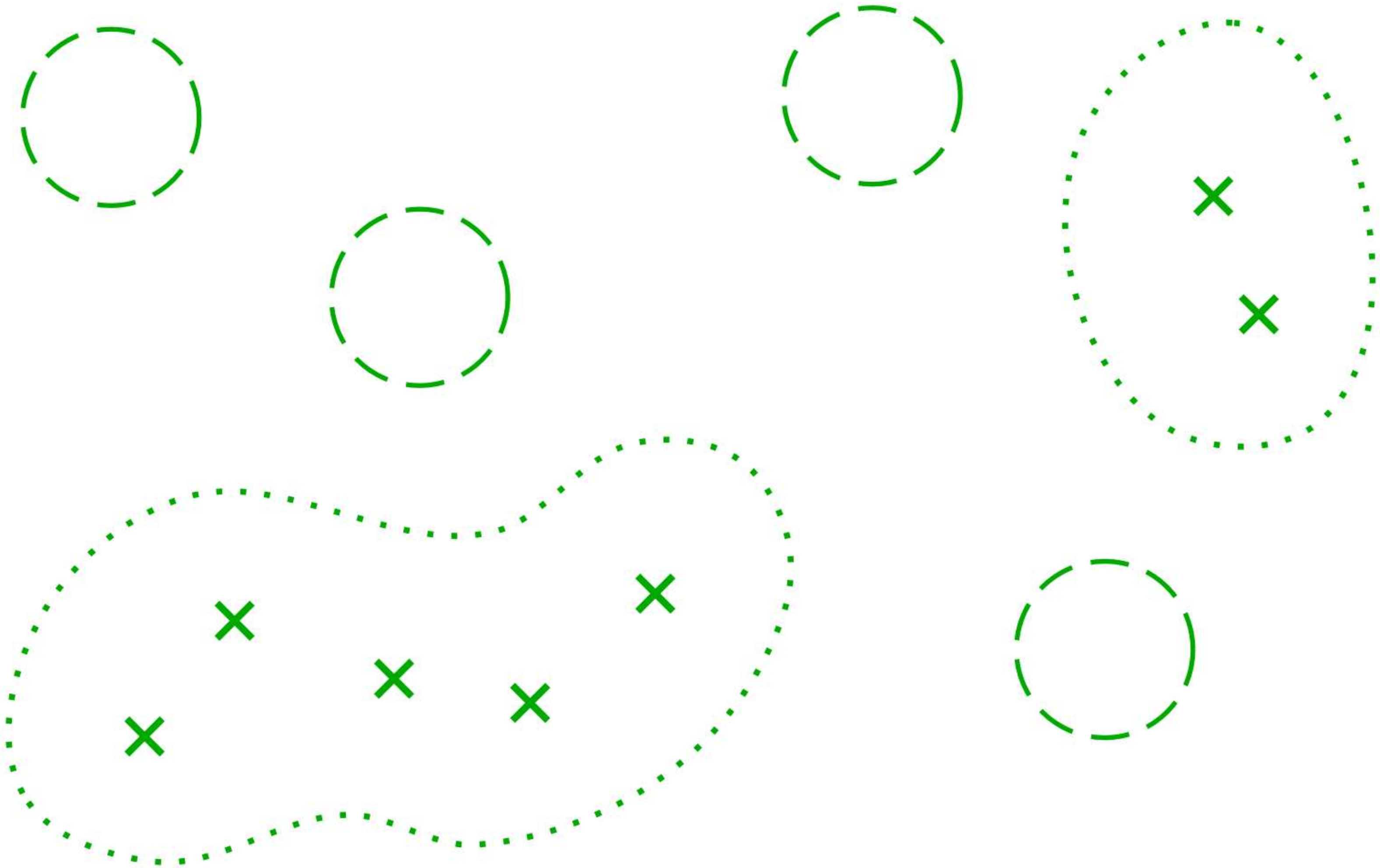
APX-hard $\Rightarrow$ no QPTAS (unless QP=NP)

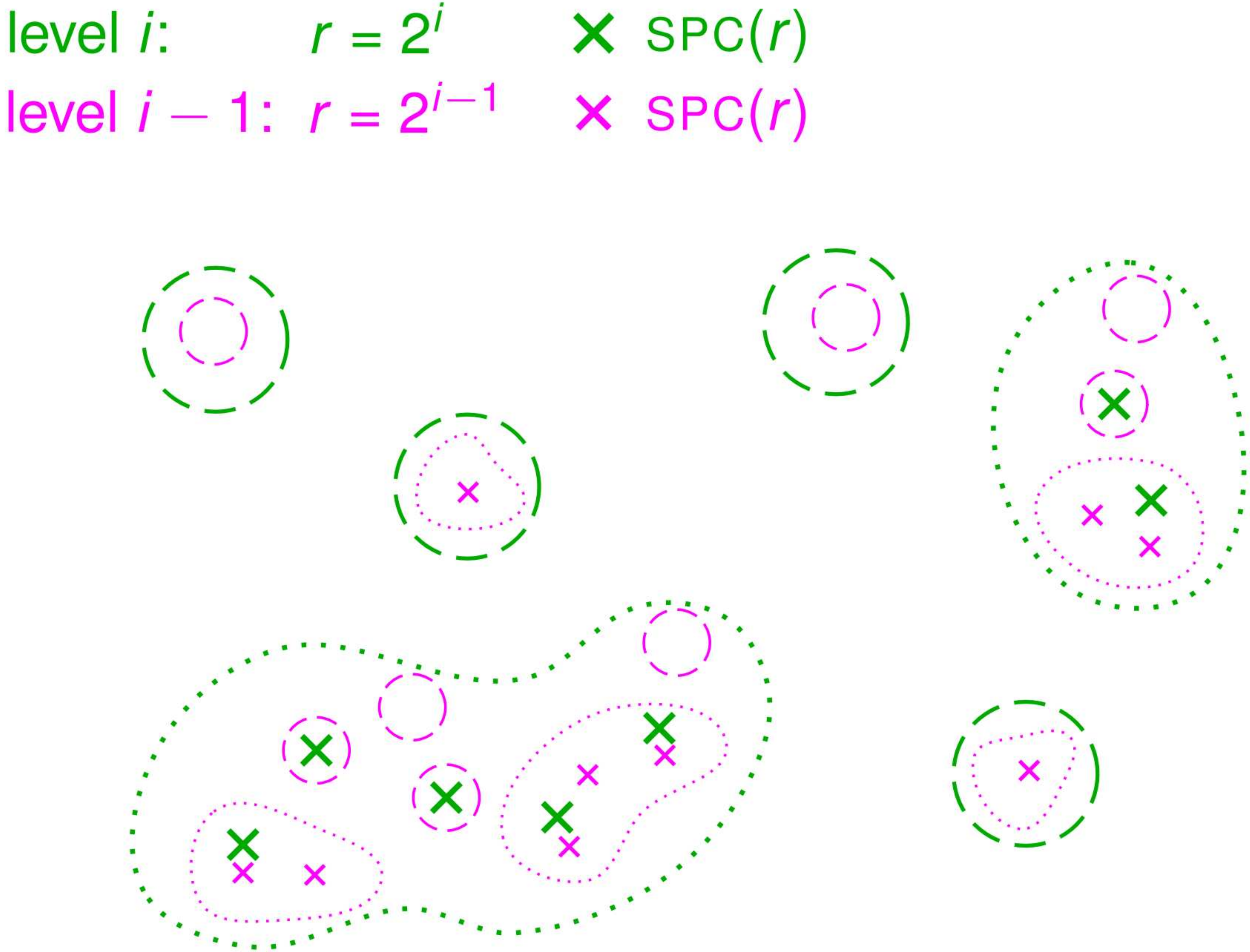

SPC(r)

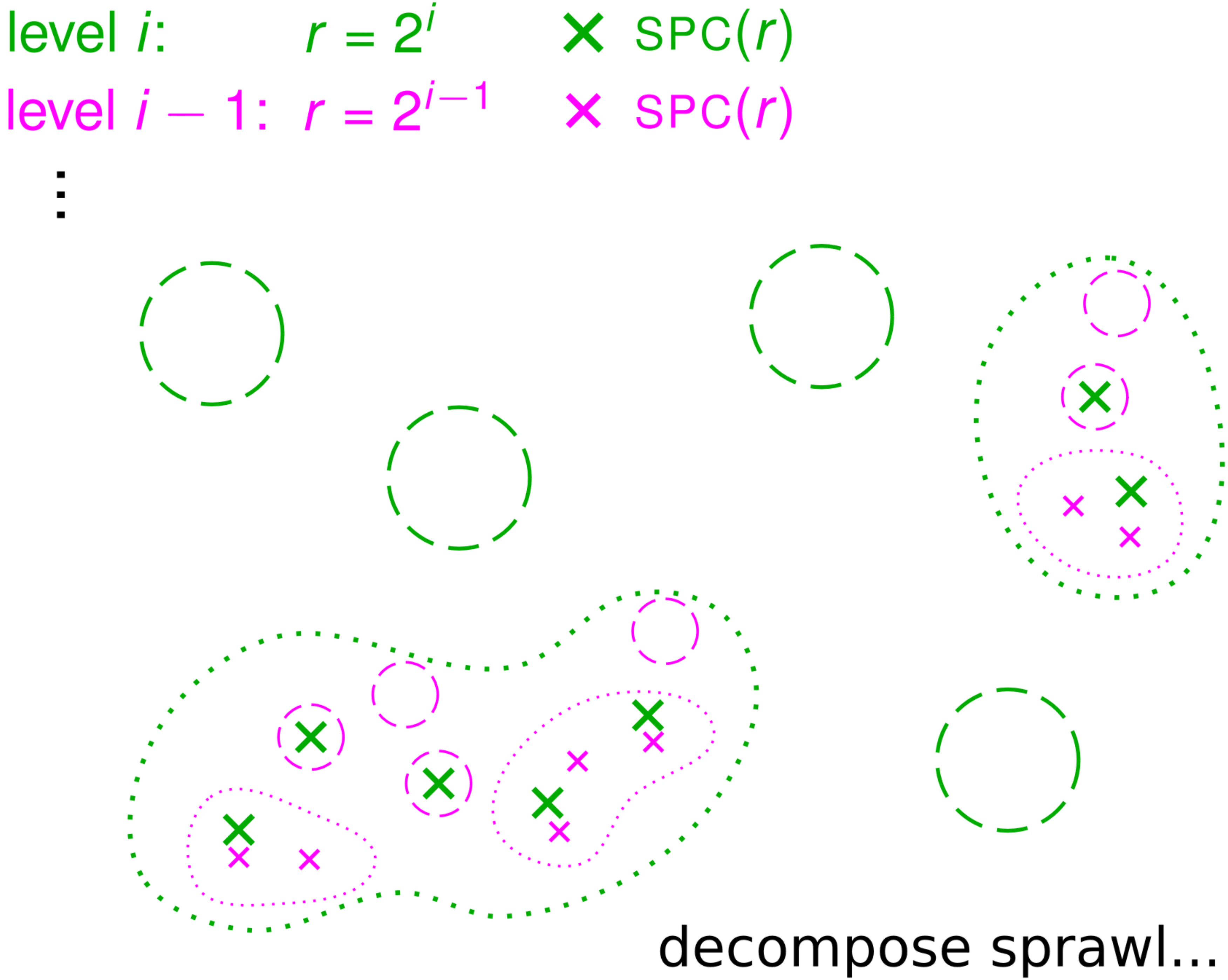

SPC(r)
towns
sprawl
$2r$


SPC(r)
towns
$>2r$
$\leq r$
$>r$
sprawl
$2r$

level i: $r = 2^i$ × SPC(r)

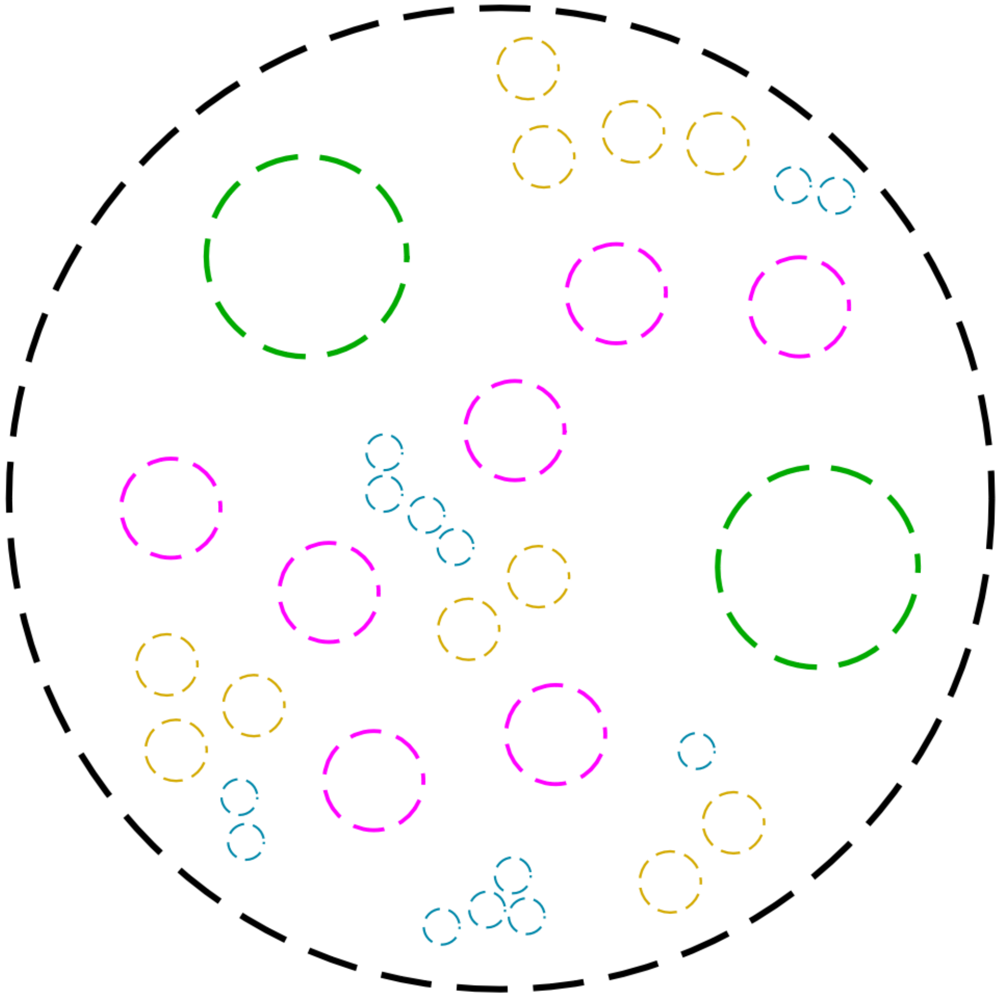



level i: $r = 2^i$ SPC(r)
level $i-1$: $r = 2^{i-1}$ SPC(r)

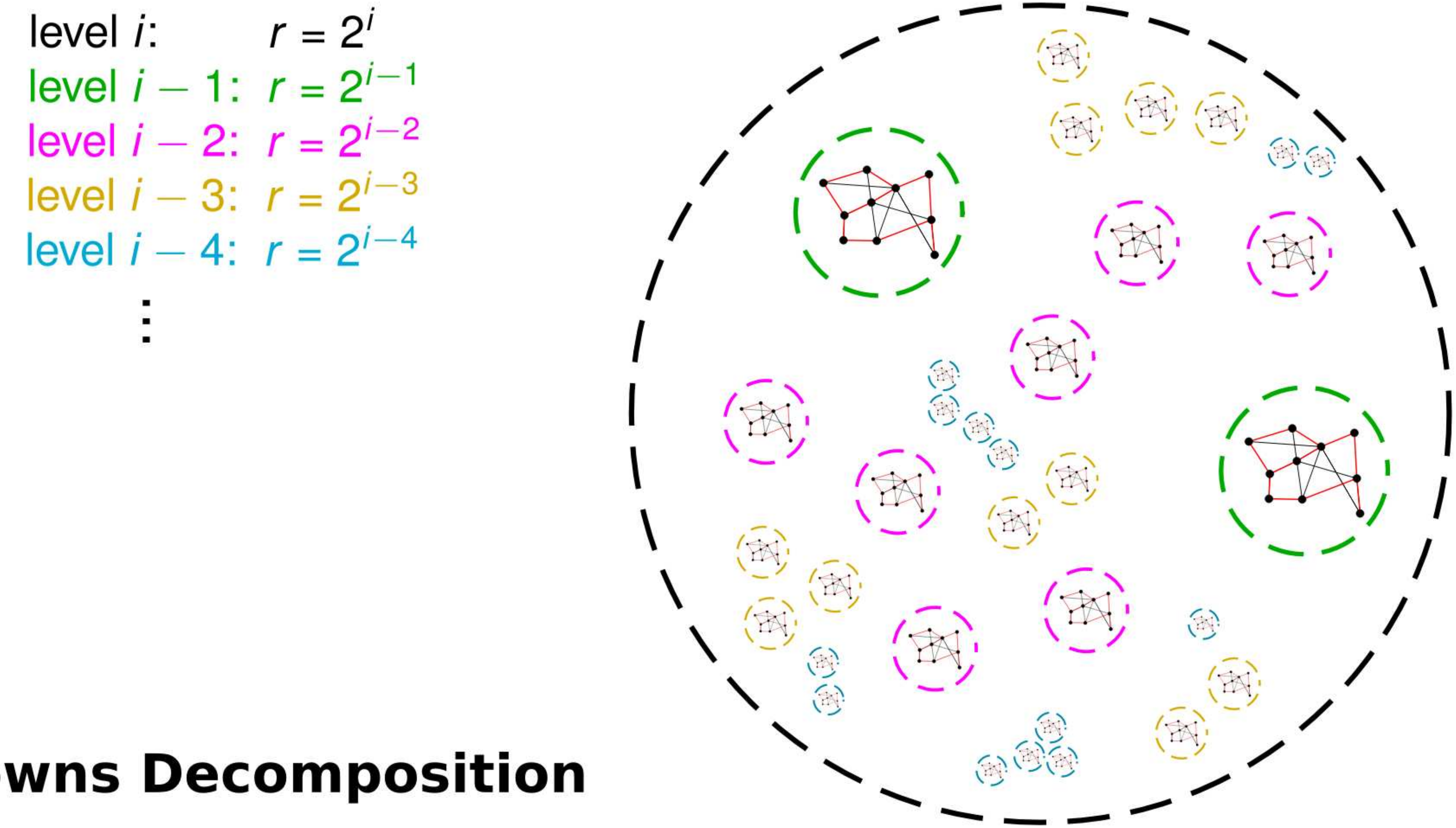


level i: $r = 2^i$ SPC(r)
level $i-1$: $r = 2^{i-1}$ SPC(r)
⋮
decompose sprawl...

level i: $r = 2^{i}$
level $i-1$: $r = 2^{i-1}$
level $i-2$: $r = 2^{i-2}$
level $i-3$: $r = 2^{i-3}$
level $i-4$: $r = 2^{i-4}$
⋮

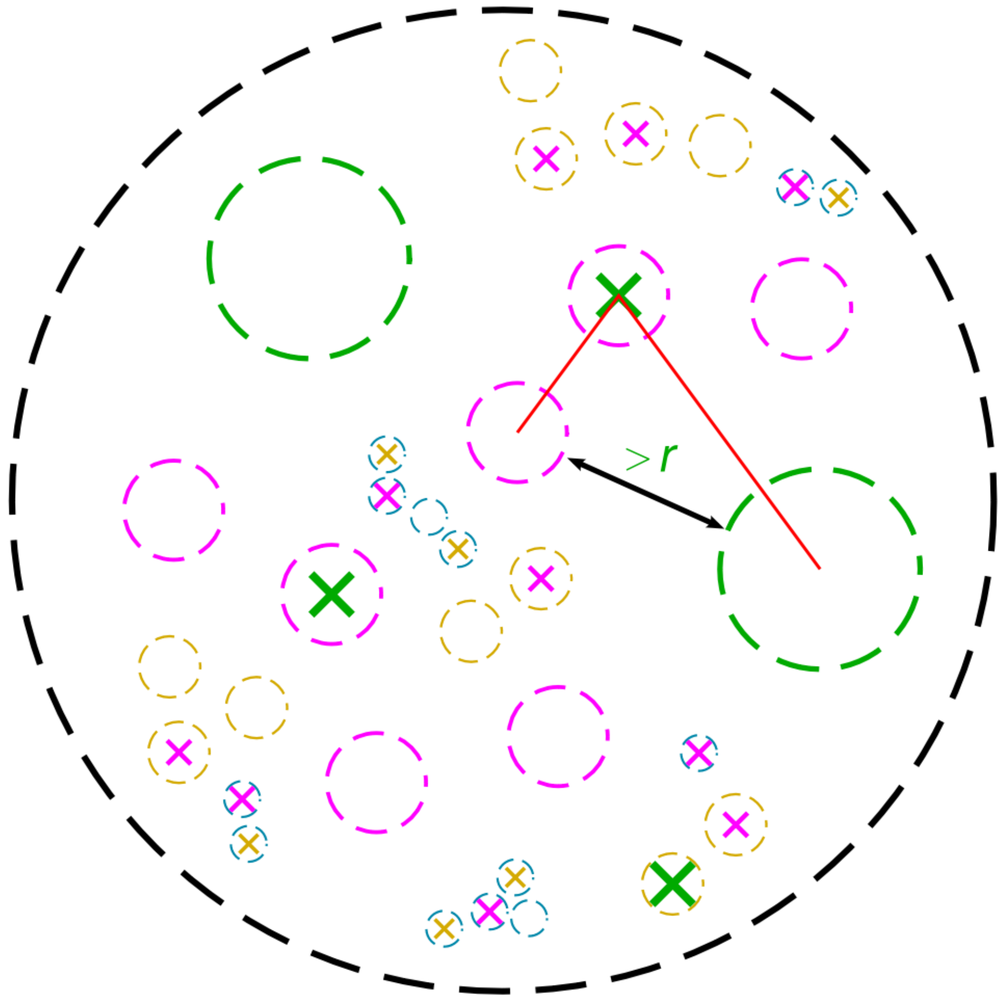


Towns Decomposition



Towns Decomposition

level i: $r = 2^{i}$
level $i - 1$: $r = 2^{i-1}$
level $i - 2$: $r = 2^{i-2}$
level $i - 3$: $r = 2^{i-3}$
level $i - 4$: $r = 2^{i-4}$
⋮

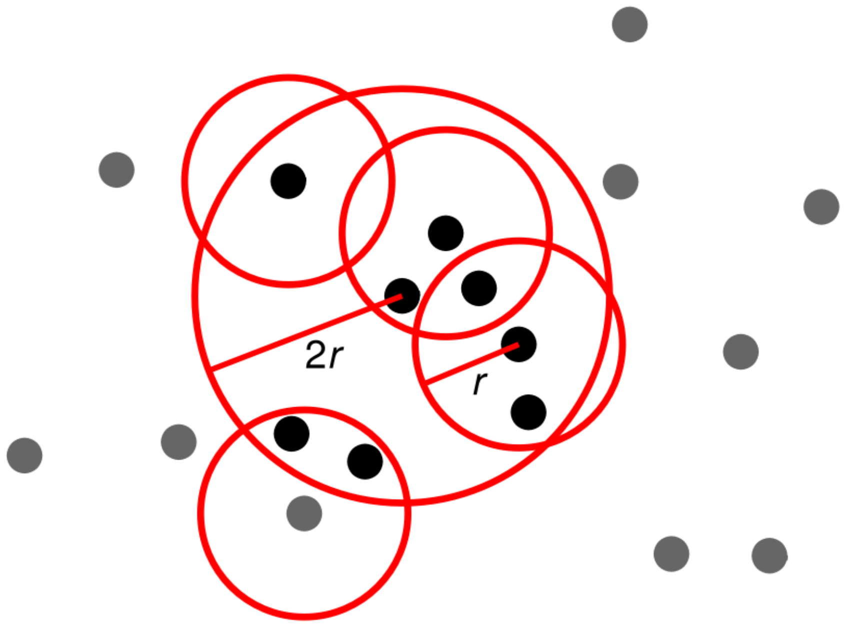


Core Hubs

Doubling dimension d:

for every ball $B_{2r}(v) := \{w \in V \mid \text{dist}(v, w) \leq 2r\}$
there are at most 2^d balls of radius r covering $B_{2r}(v)$.

"bounded volume growth":
Euclidean/Manhattan metrics



Theorem

[Talwar '04]

For any $\varepsilon > 0$ and any graph with

- doubling dimension d
- aspect ratio α

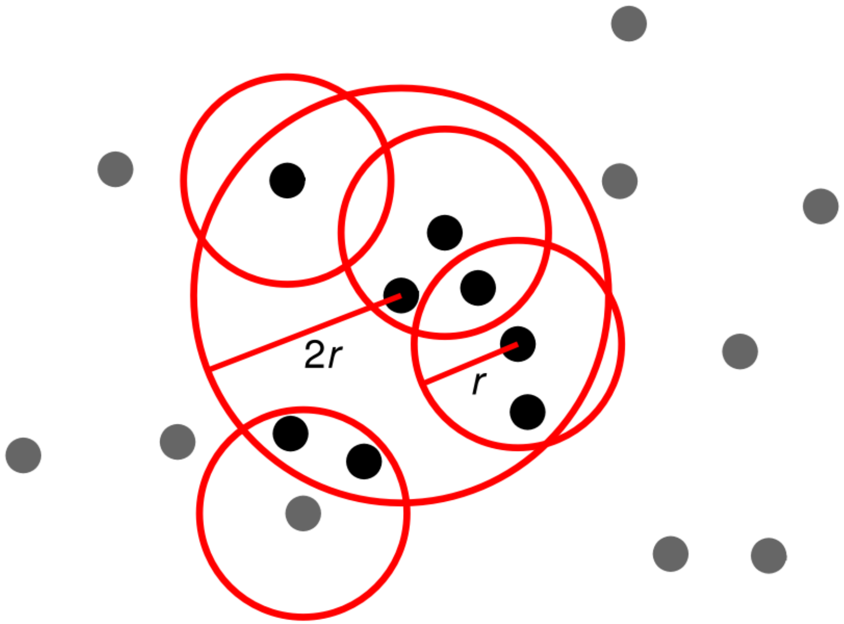
there is a polynomial-time computable probabilistic embedding with

1. small distortion of distances:

   expected distortion $1 + \varepsilon$

2. structurally simple:

   treewidth $(d \log(\alpha)/\varepsilon)^{O(d)}$



level i: $r = 2^i$
level $i-1$: $r = 2^{i-1}$
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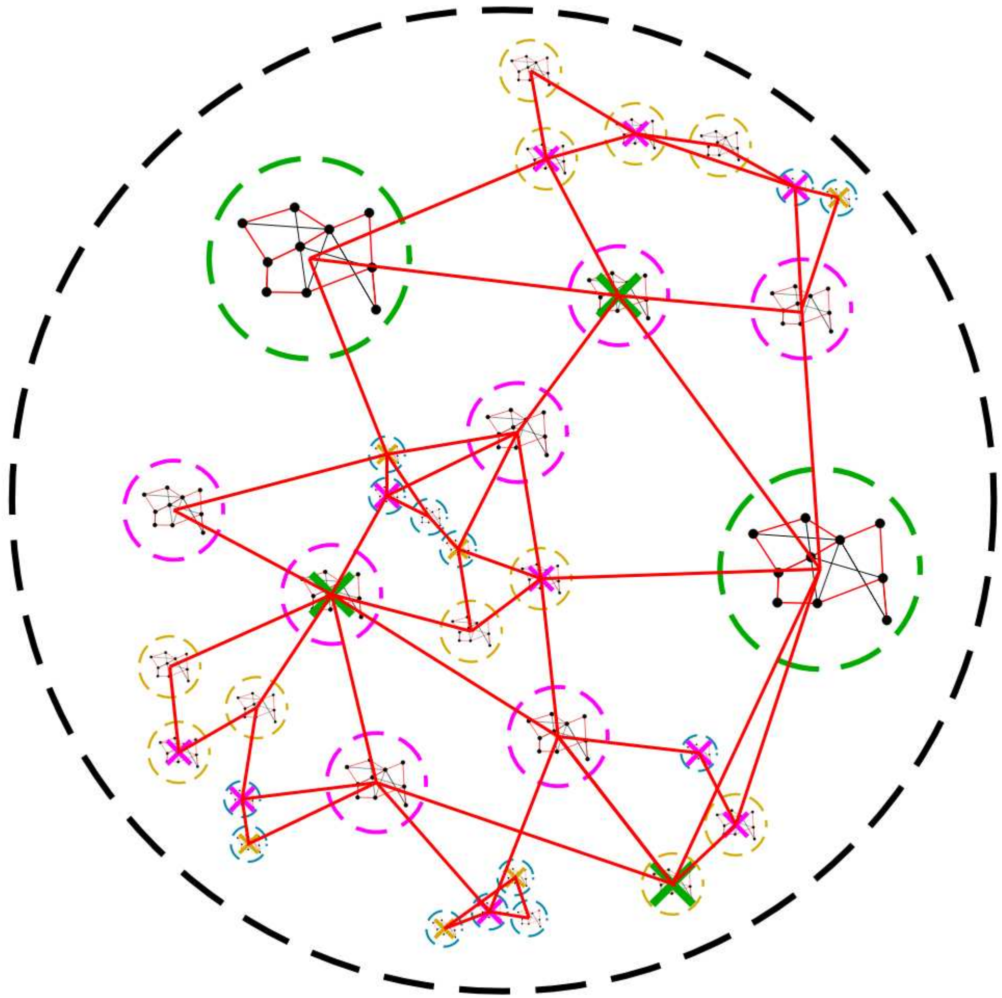
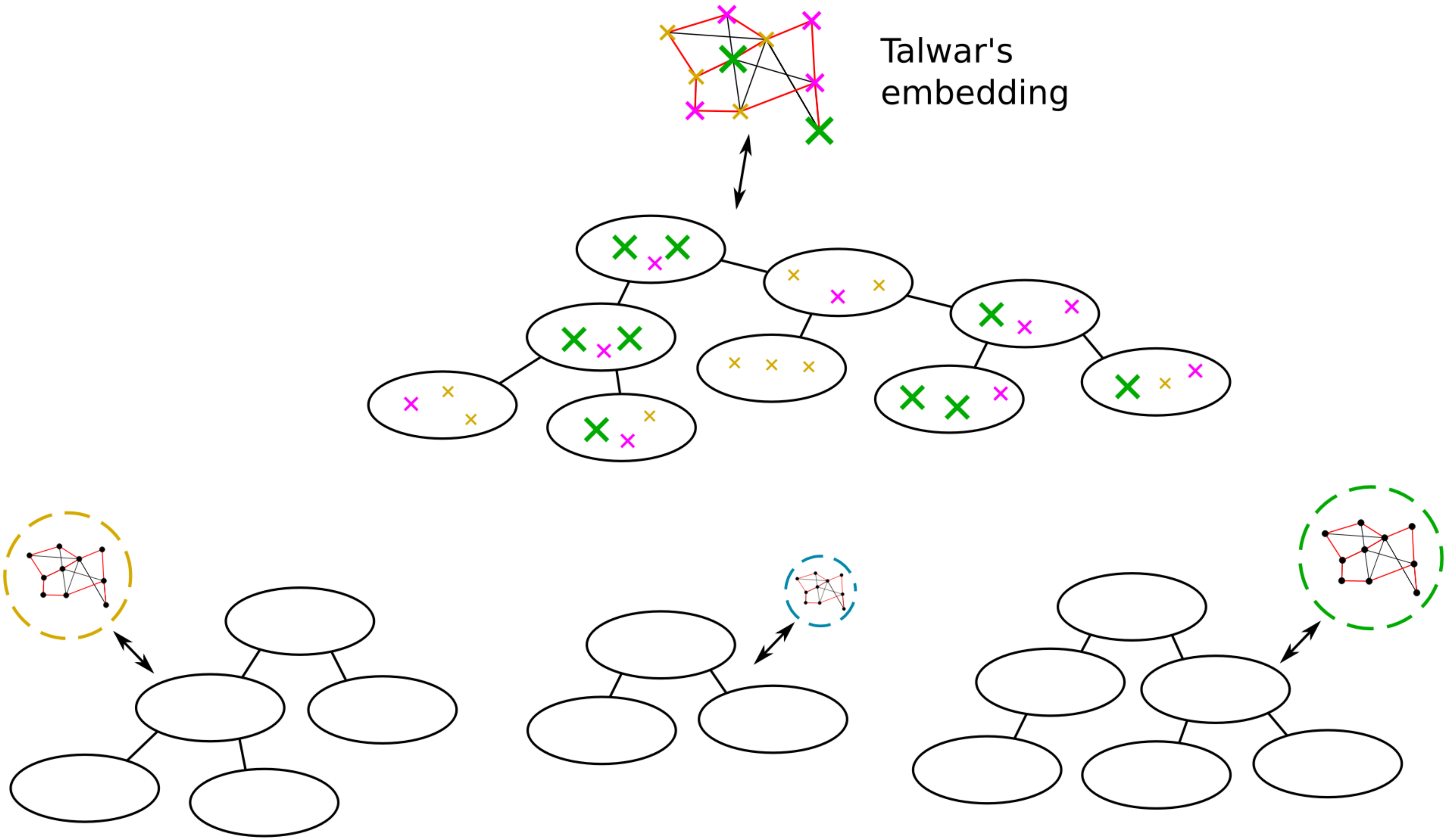


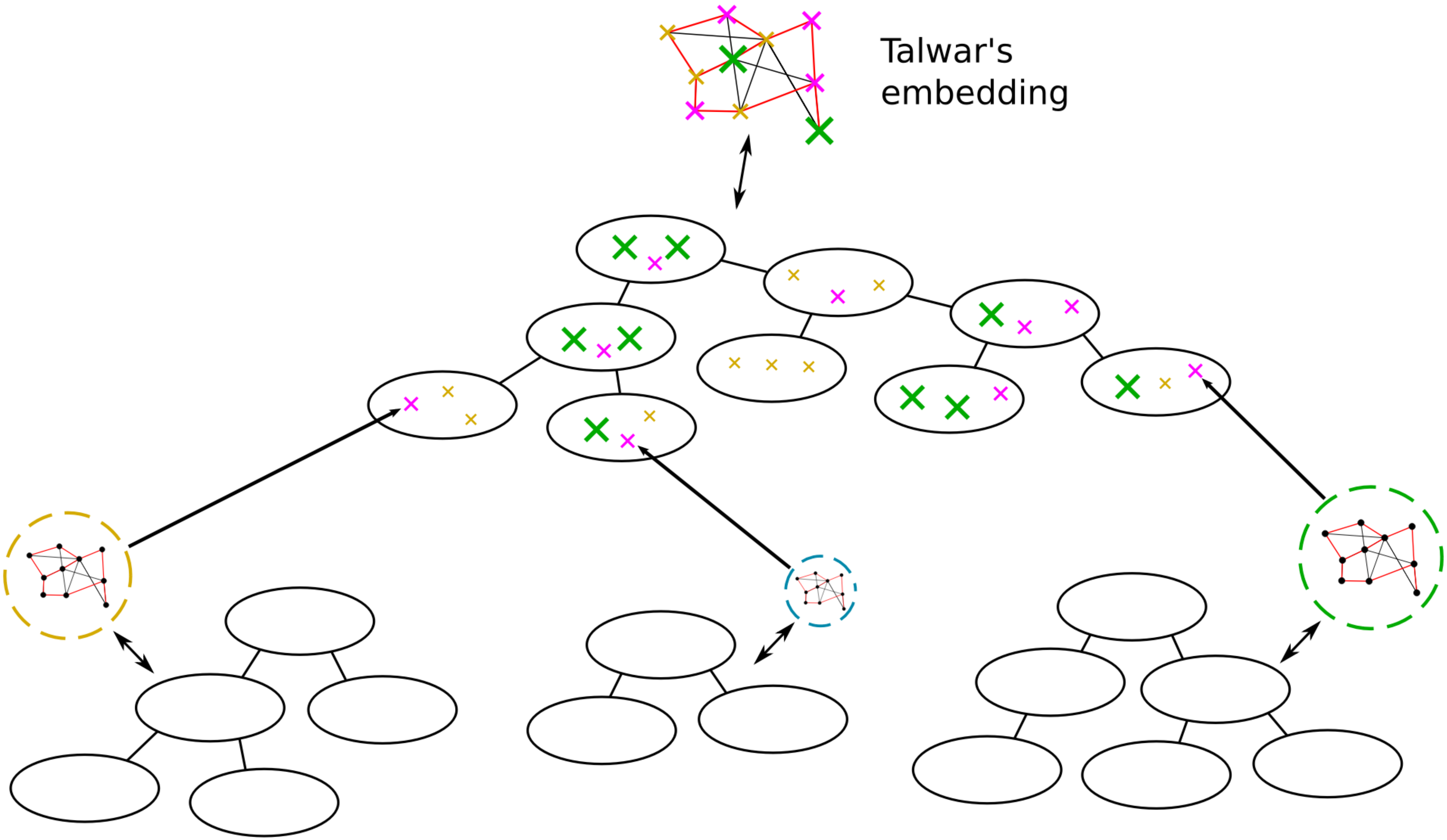
Core Hubs

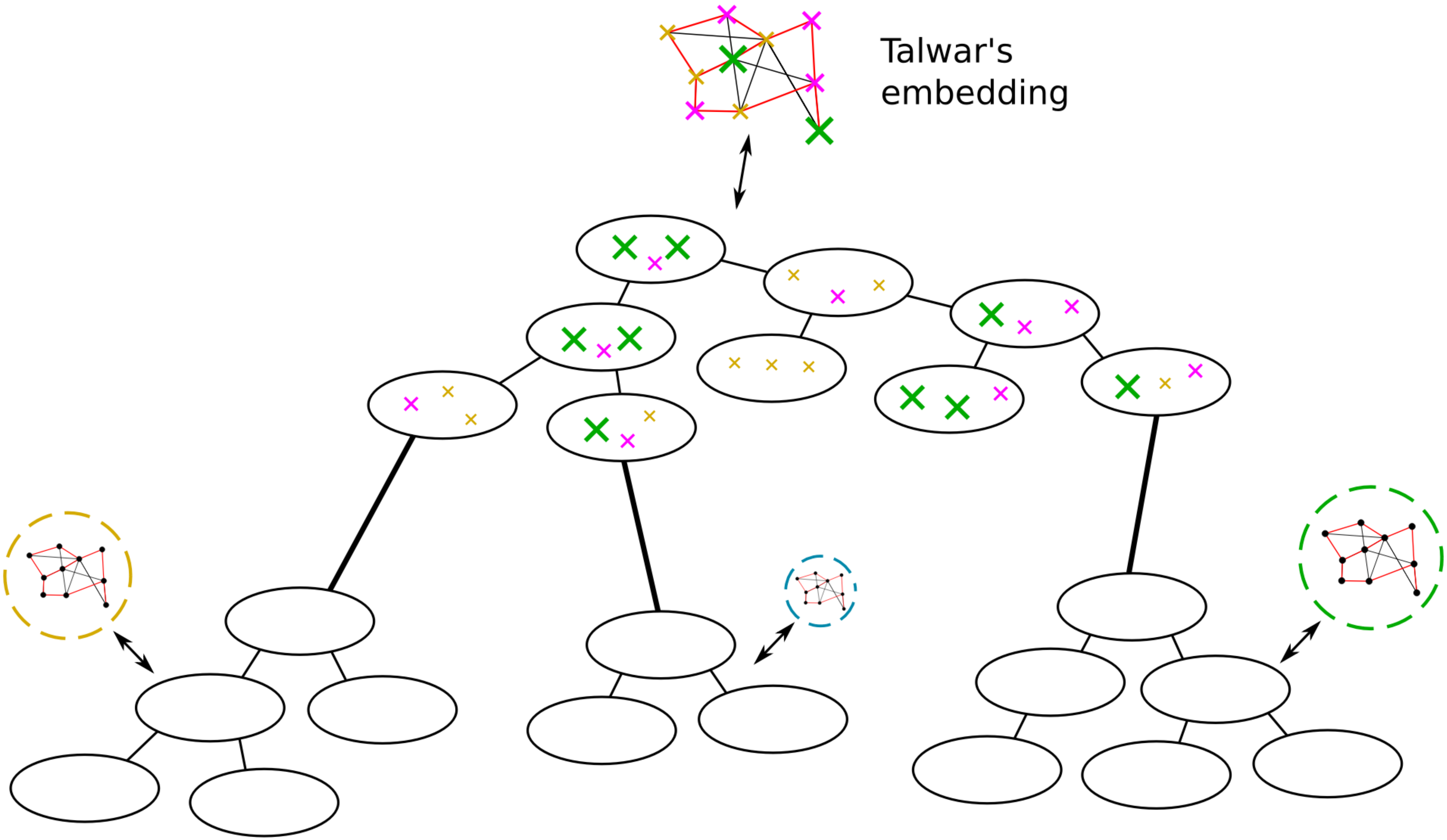
low doubling dimension

→ Talwar's embedding



Talwar's
embedding


Talwar's embedding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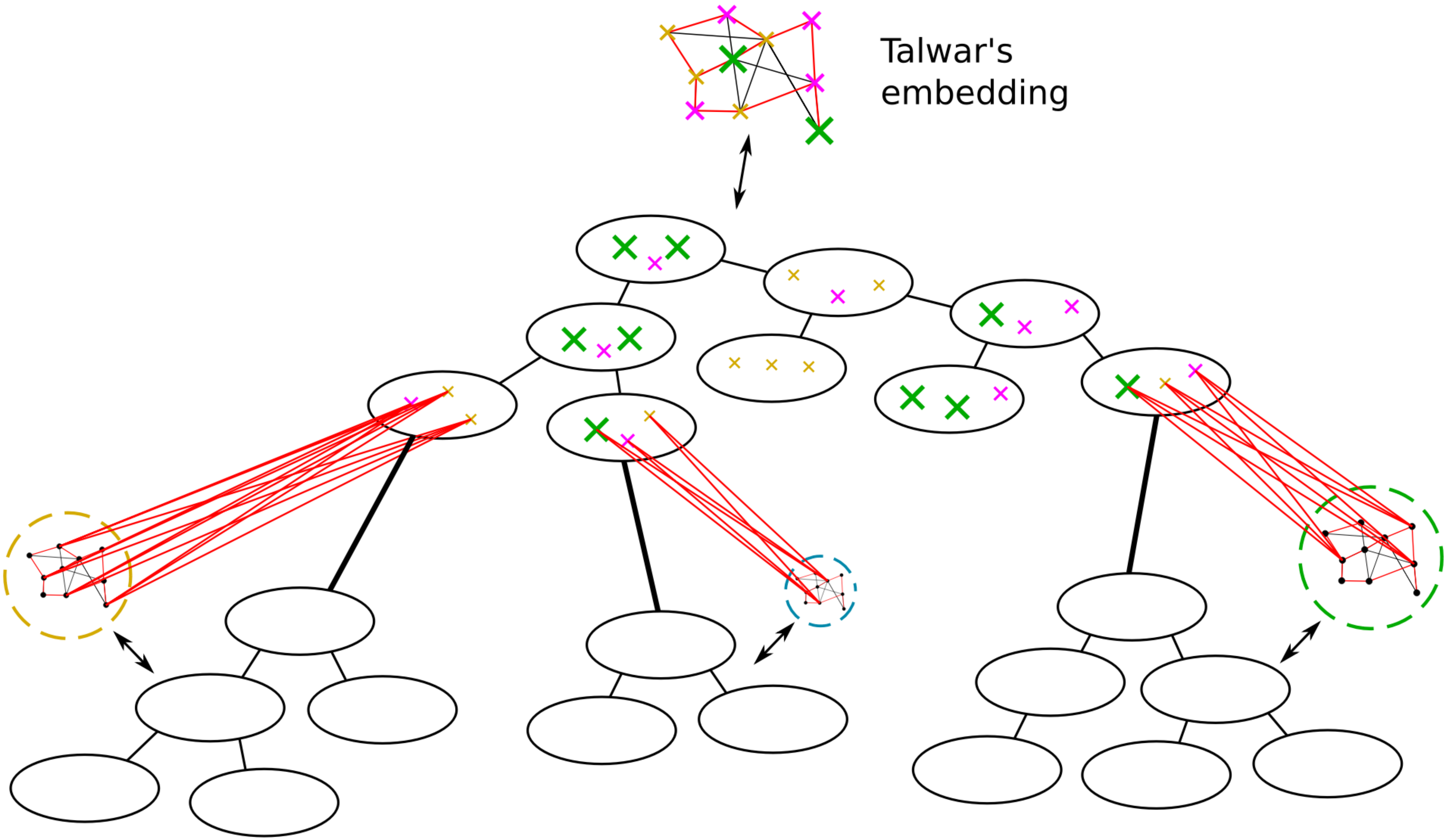
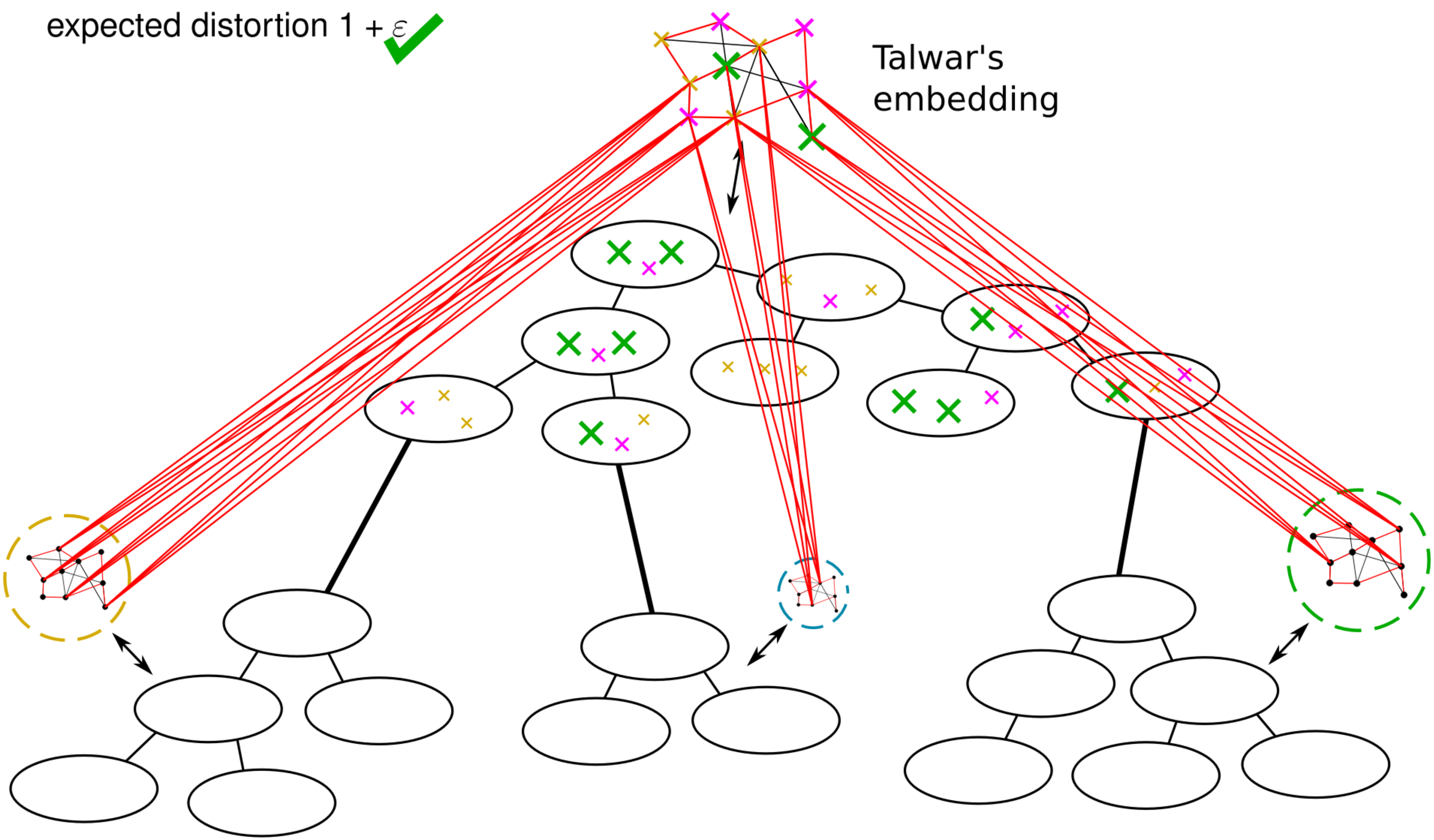
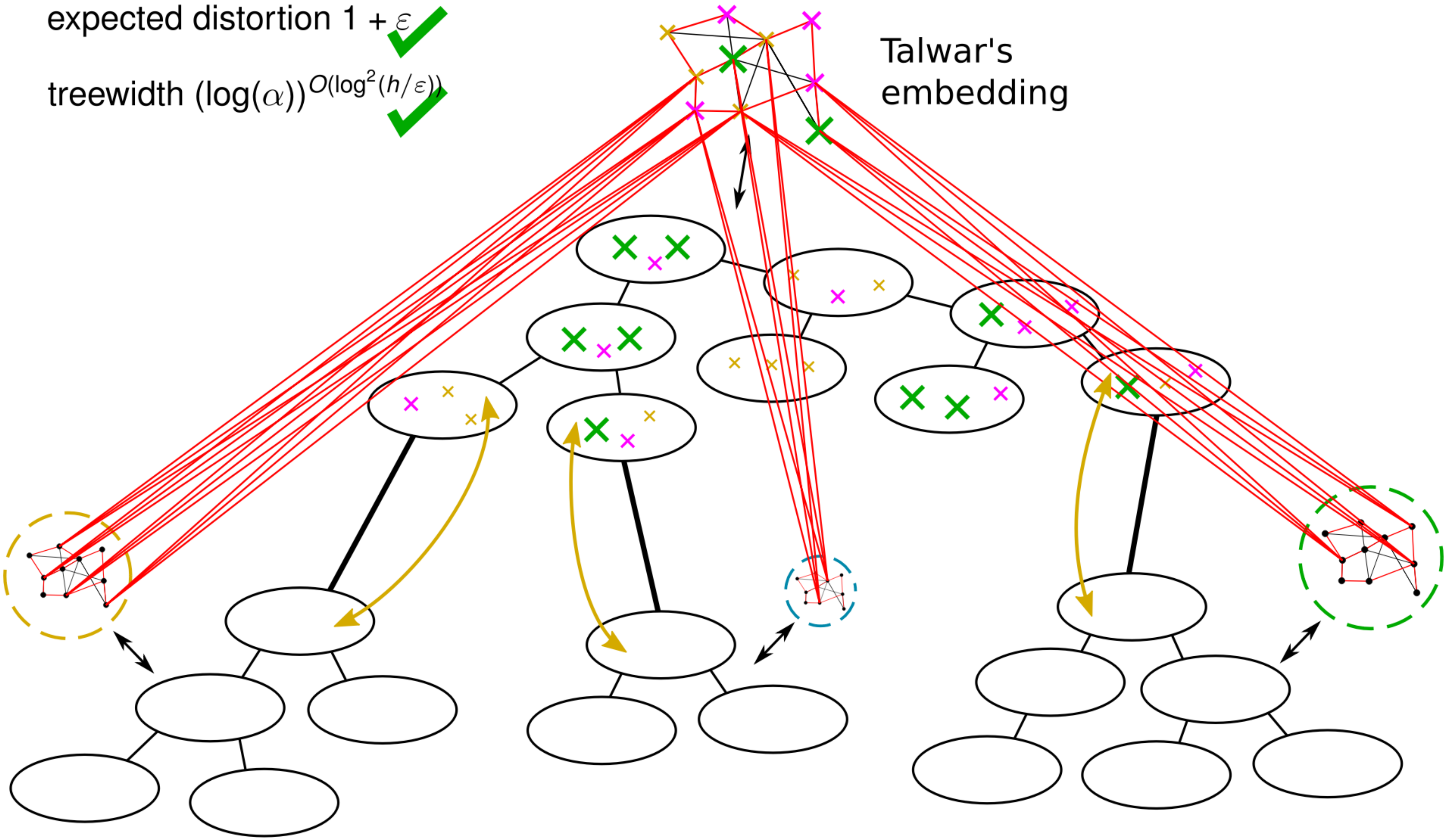
embedding


Talwar's
embedding


expected distortion 1 + ε
Talwar's
embedding


expected distortion $1+\varepsilon$
treewidth $(\log(\alpha))^{O(\log^2(h/\varepsilon))}$
Talwar's
embedding

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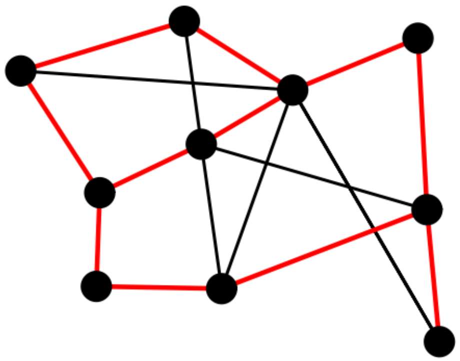

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PTAS?

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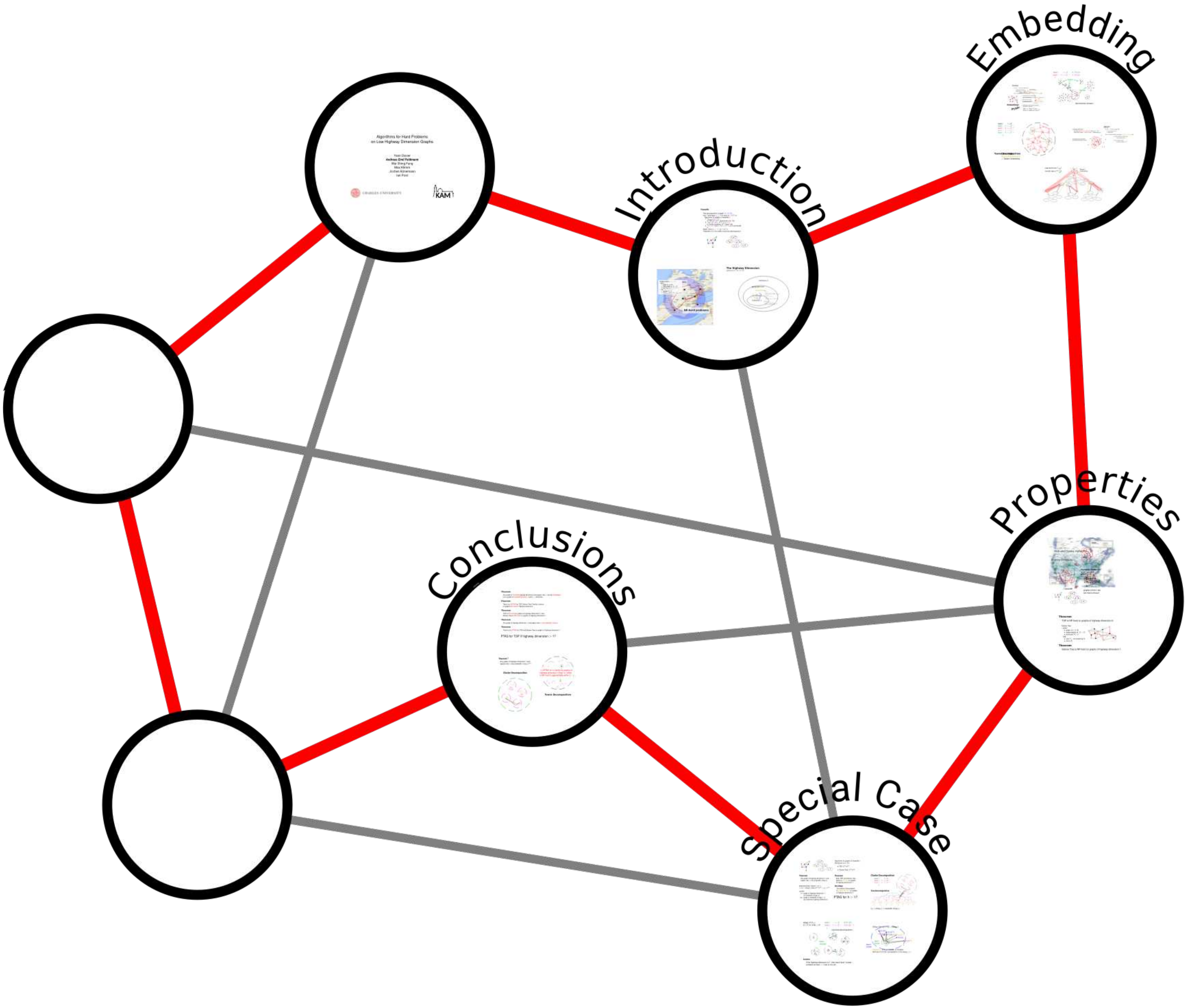

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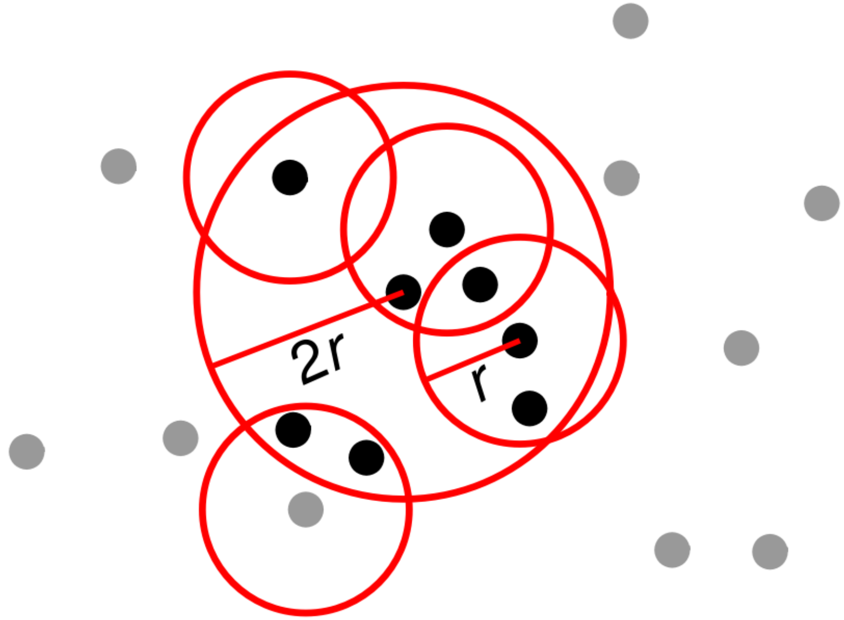
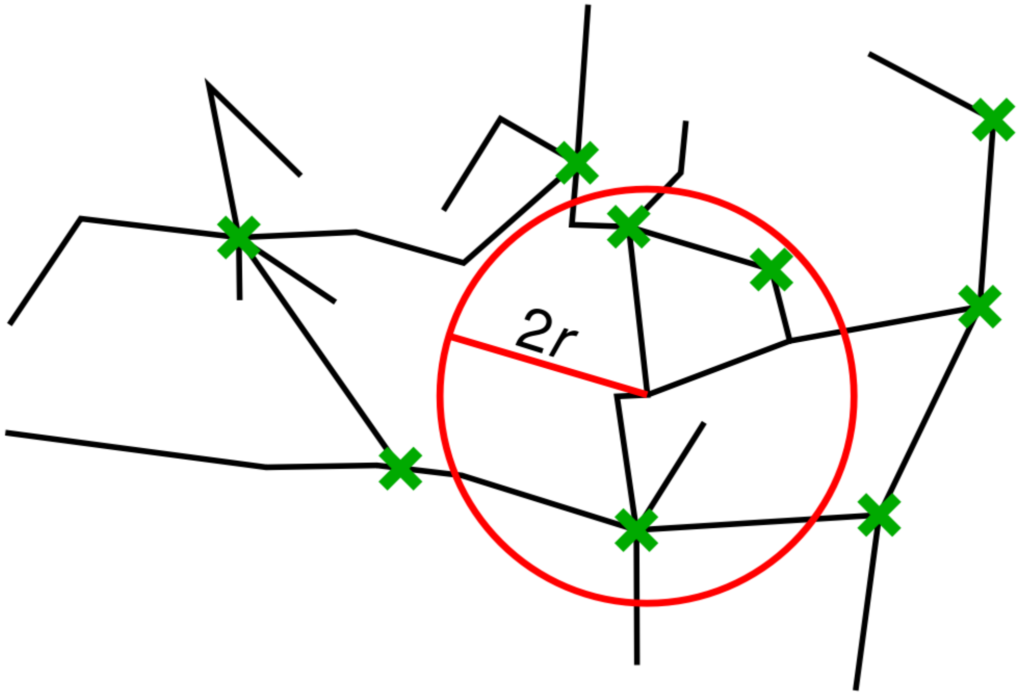
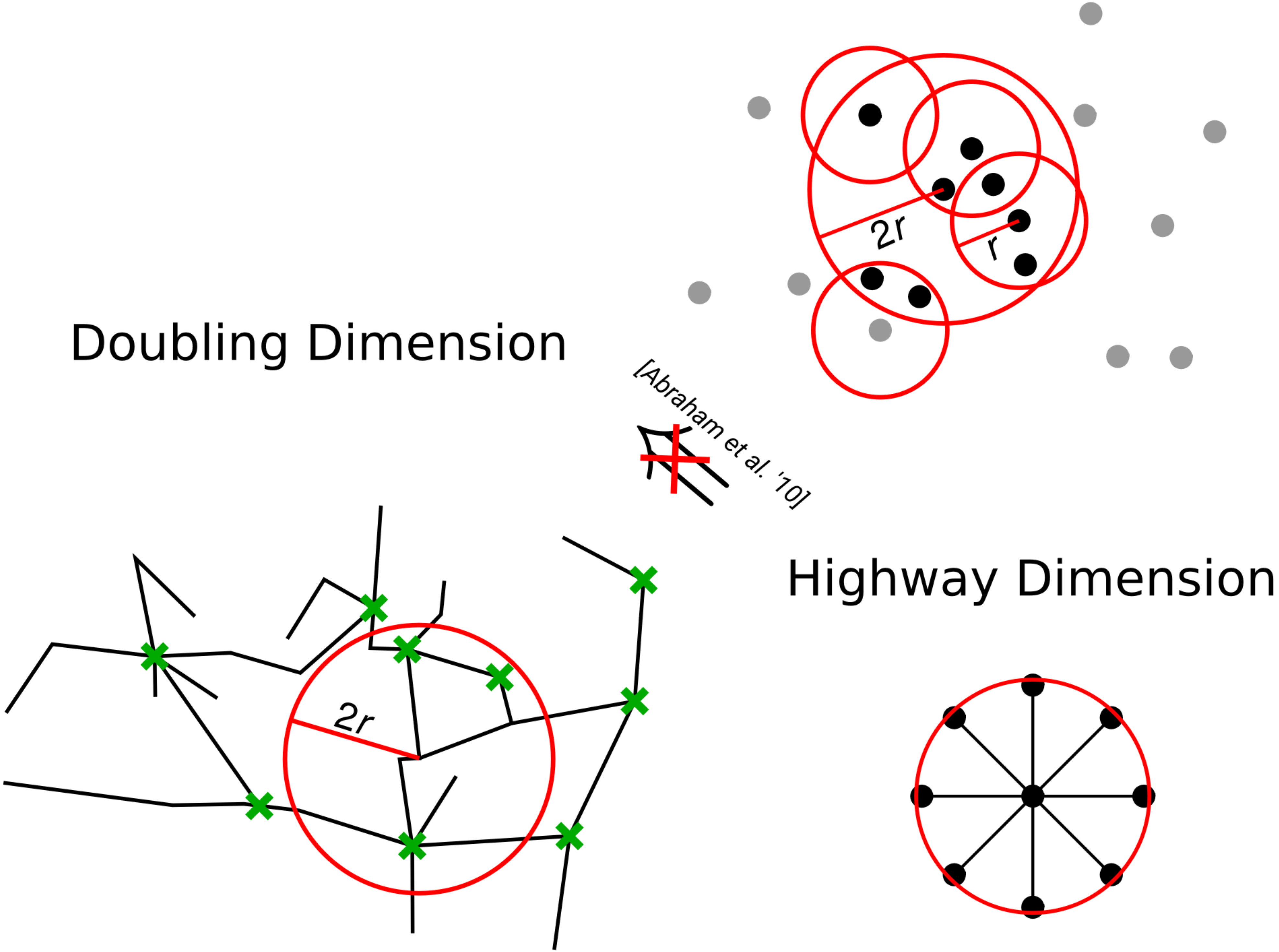


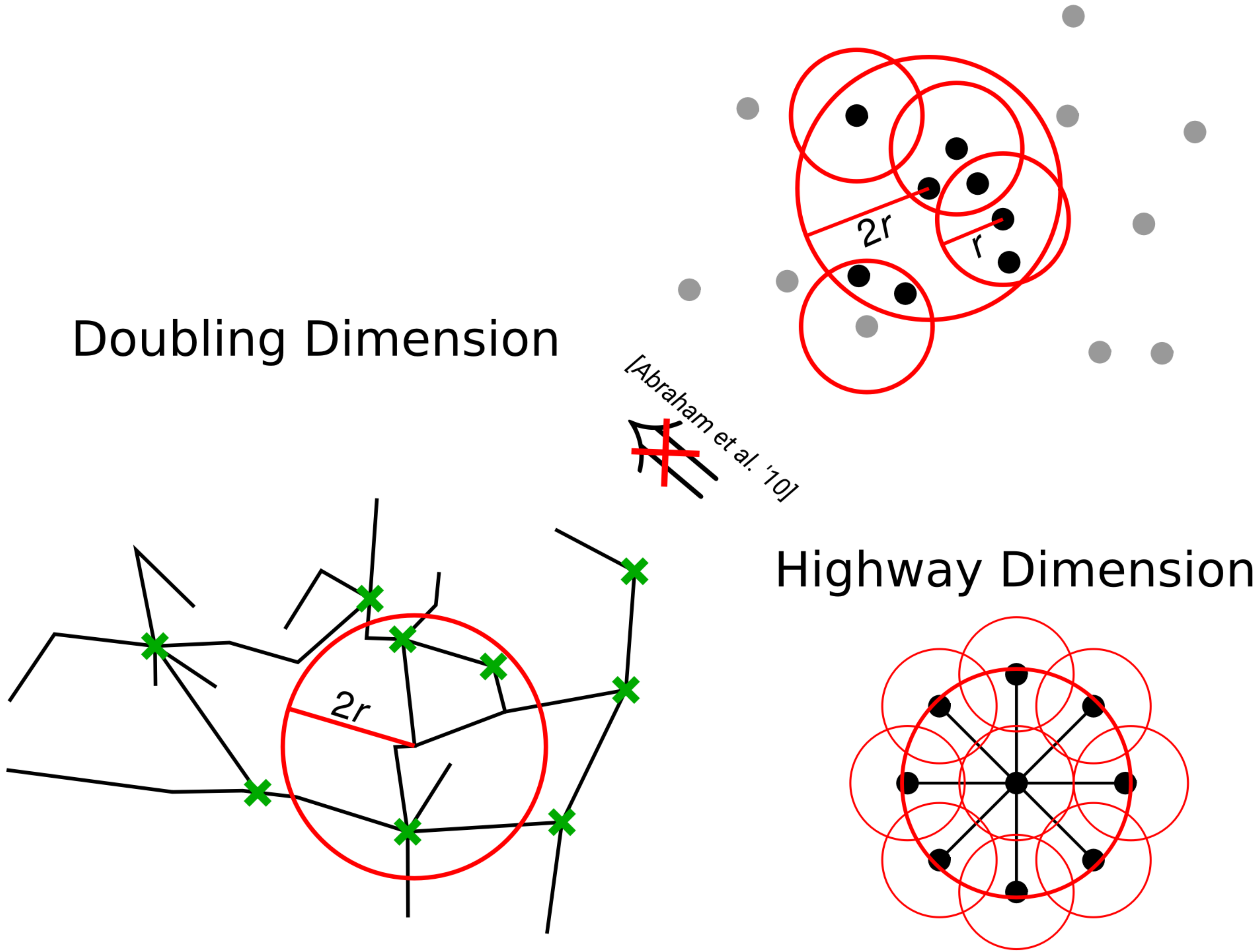
Doubling Dimension



Highway Dimension



2r
r
Doubling Dimension
[Abraham et al. '10]
Highway Dimension
2r



Doubling Dimension
2r
r
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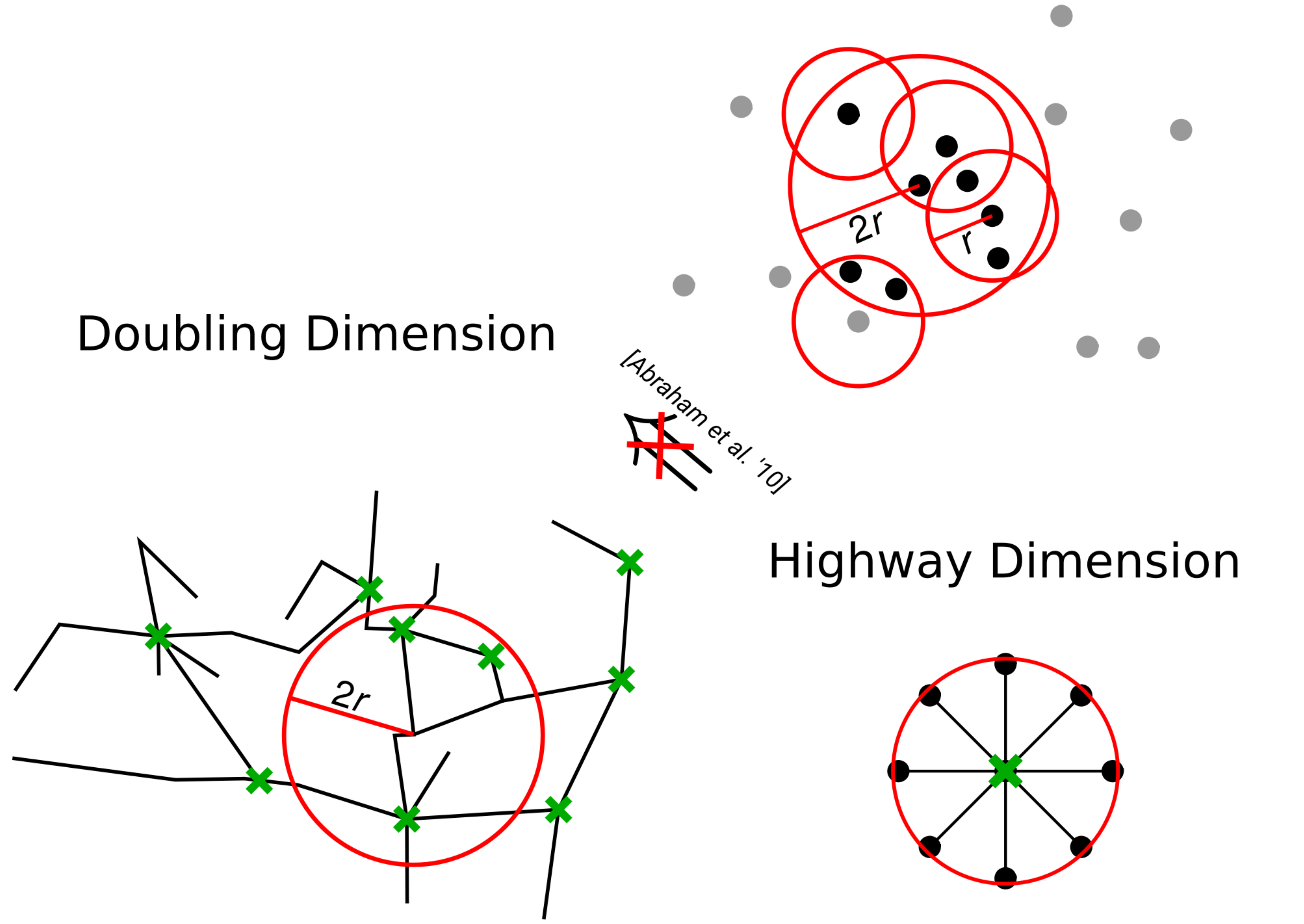
2r
r
Doubling Dimension
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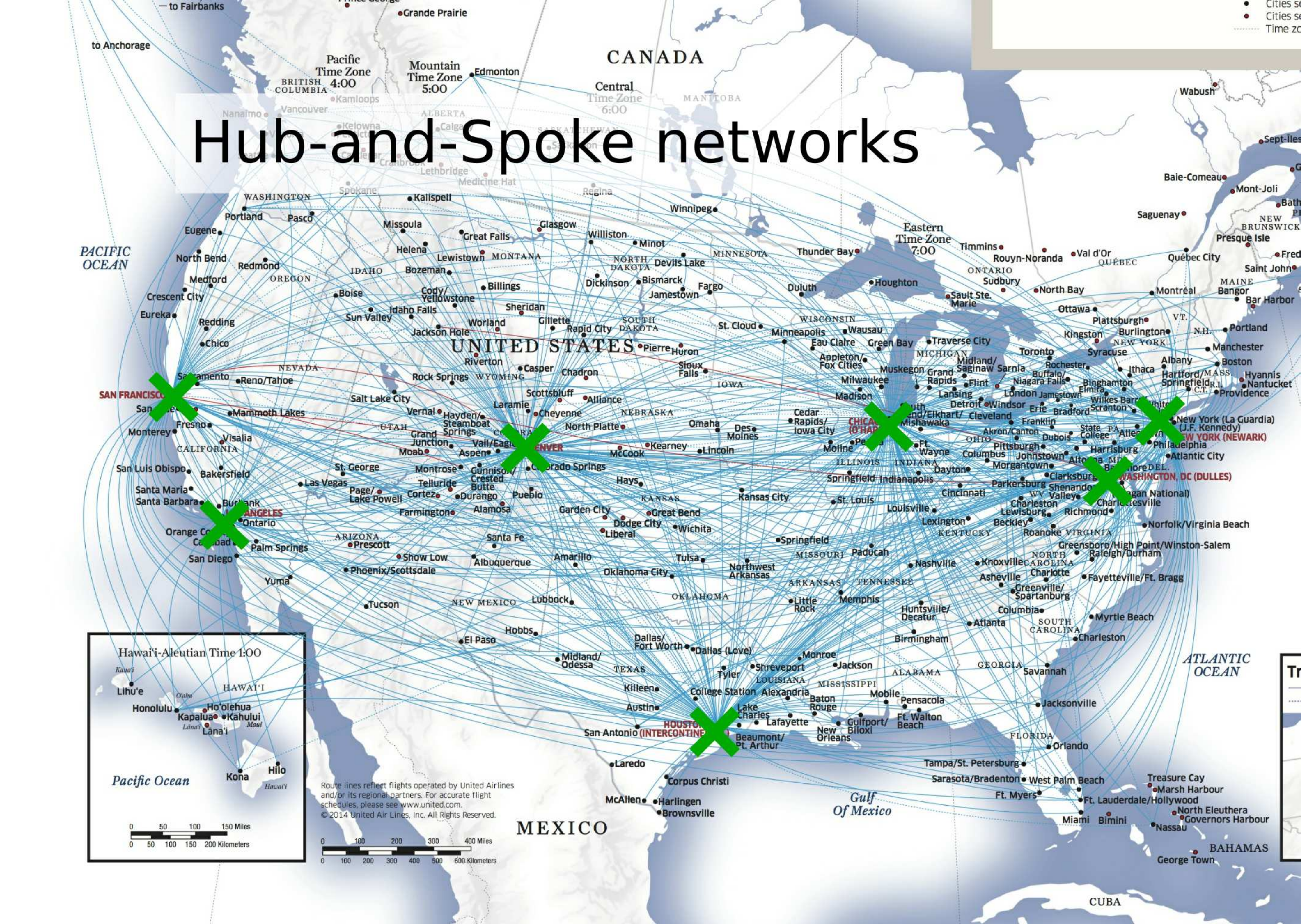
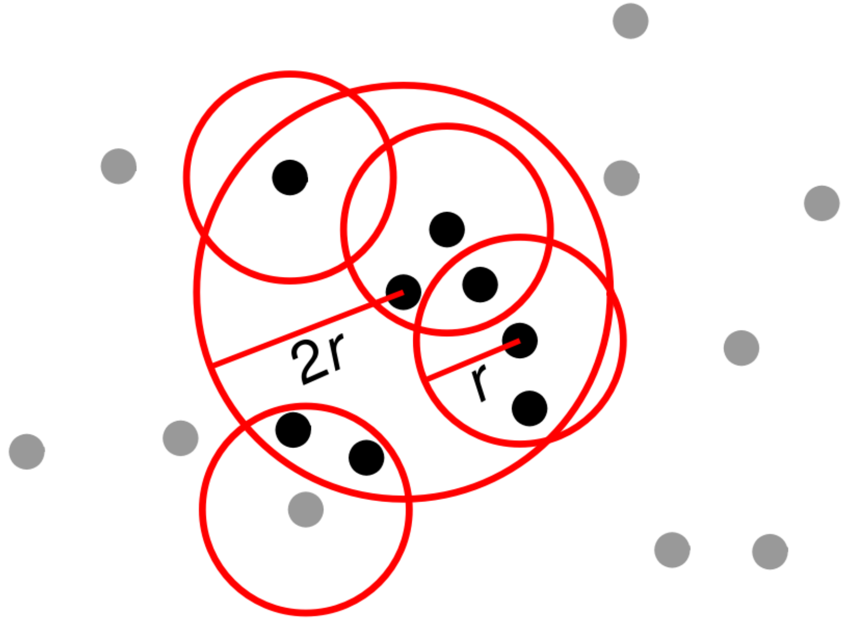
Highway Dimension
2r

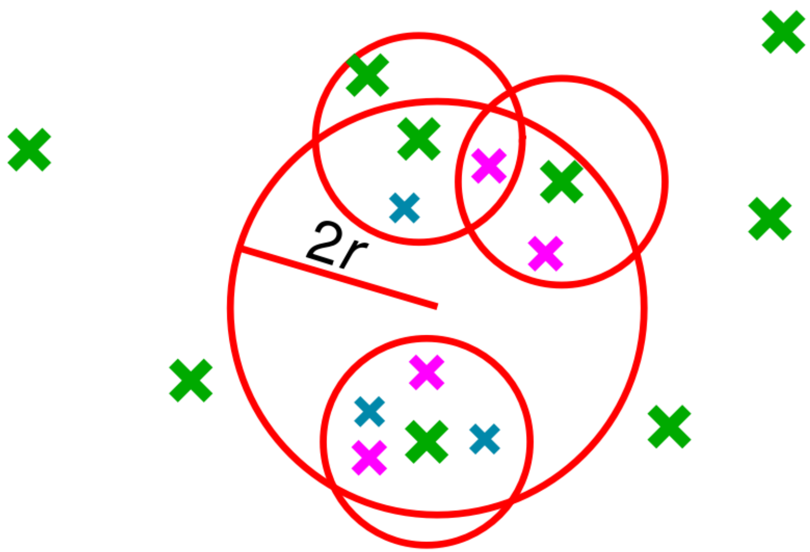
Hub-and-Spoke networks



Doubling Dimension

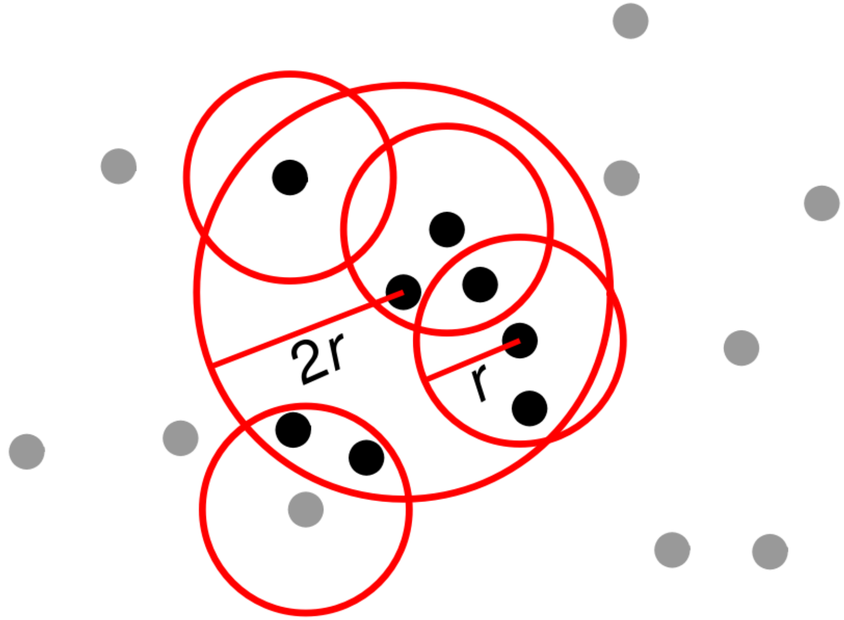


level i: $r = 2^i$ × SPC(r)



Core Hubs

Doubling Dimension



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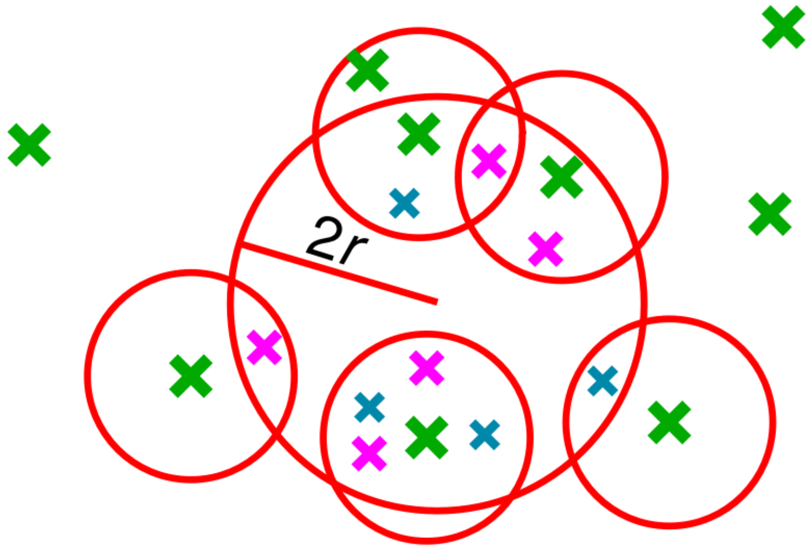
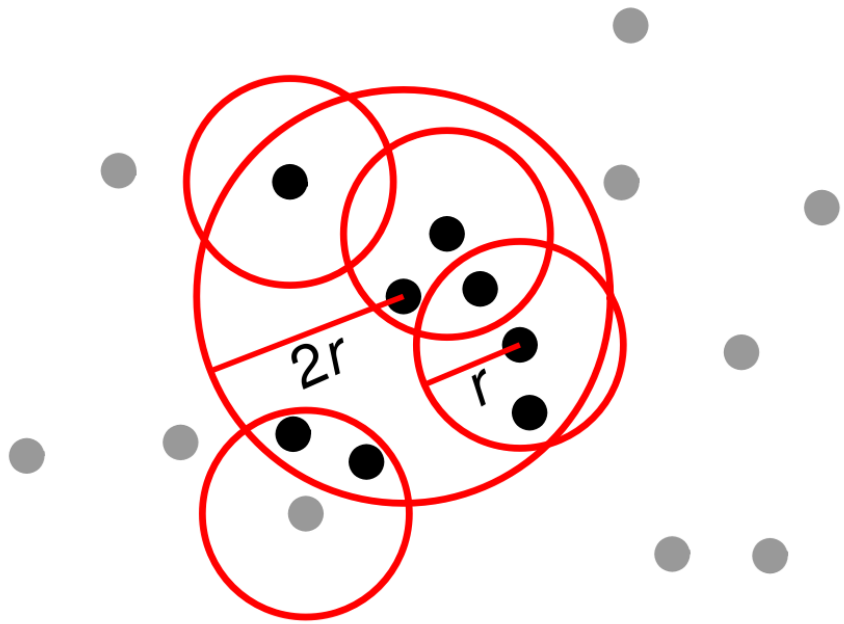
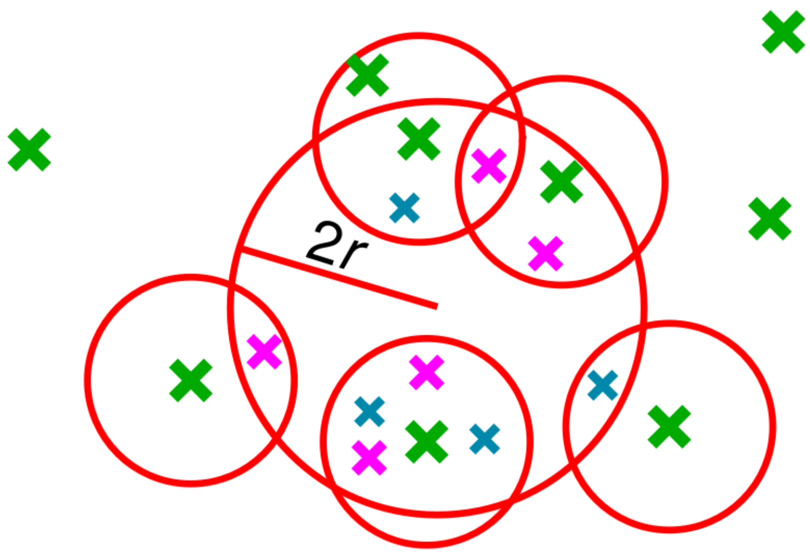


Core Hubs

Doubling Dimension



level i: $r = 2^i$ ✖ SPC(r)

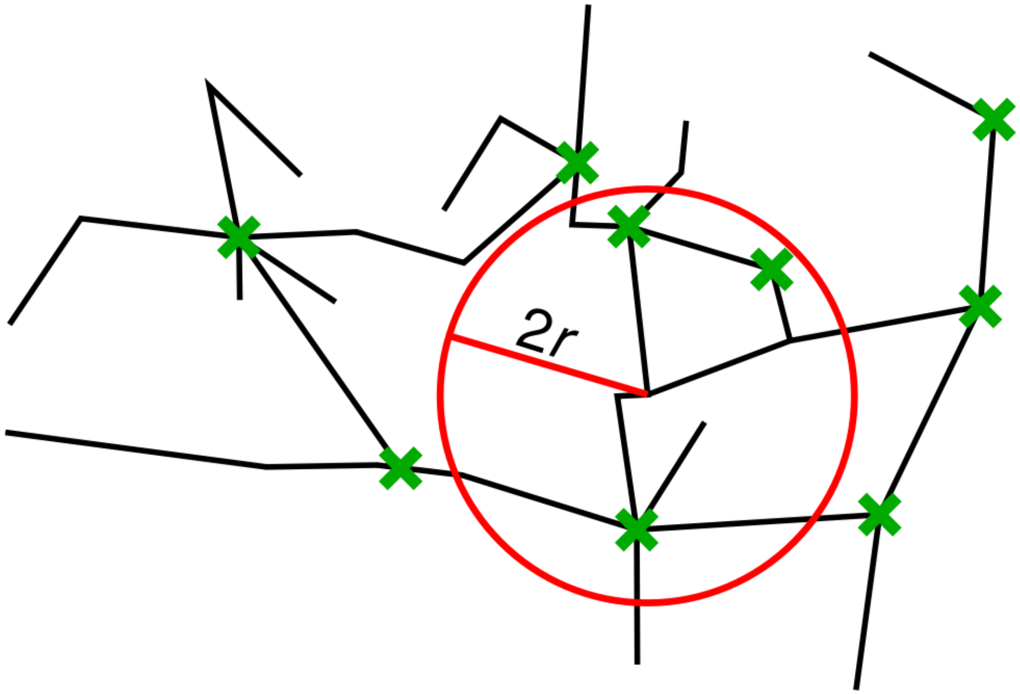


Core Hubs

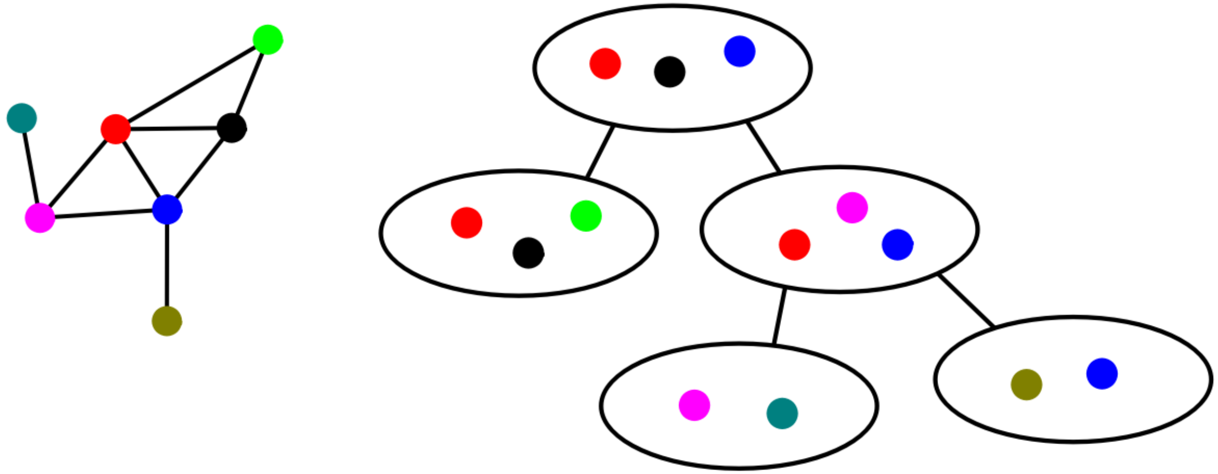
Lemma

According to Definition 2,
for any ball $B_{2r}(v)$:
$O(h^2)$ hubs $w \in \text{SPC}(r)$
s.t. $\text{dist}(w, B_{2r}(v)) \leq 2r$.

Highway Dimension

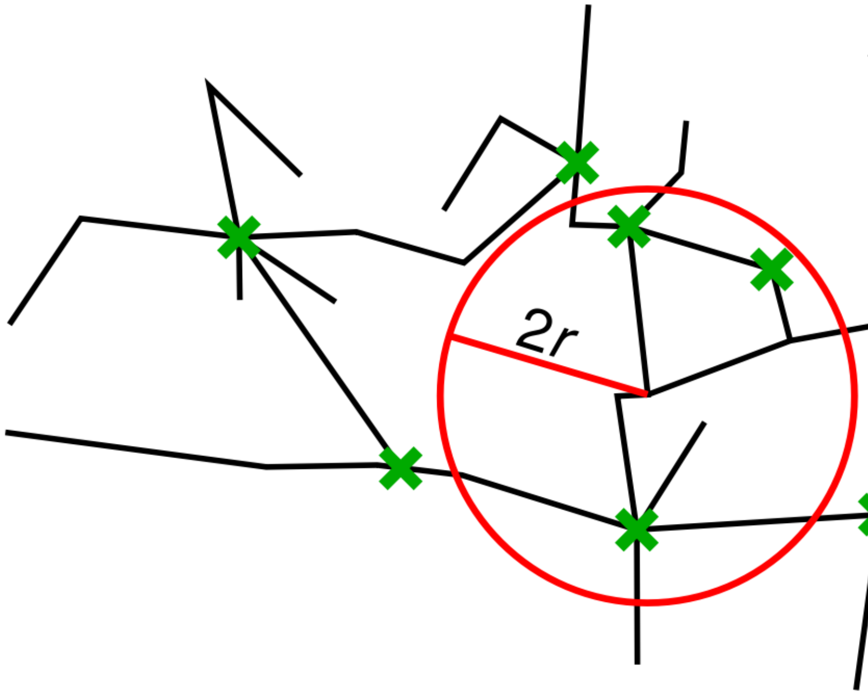


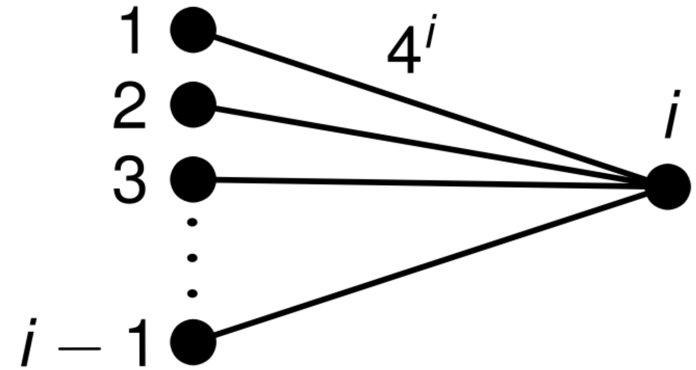
Treewidth



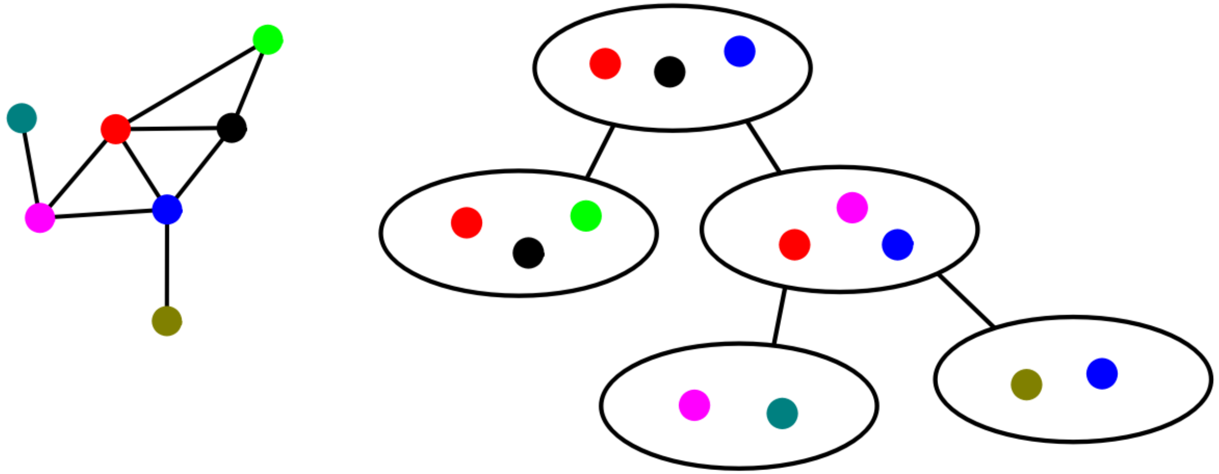
Highway Dimension

for $V = [n]$ with $E = \binom{[n]}{2}$:
$w(ij) = 4^i$ if $j < i$





Treewidth $\not\Leftarrow$



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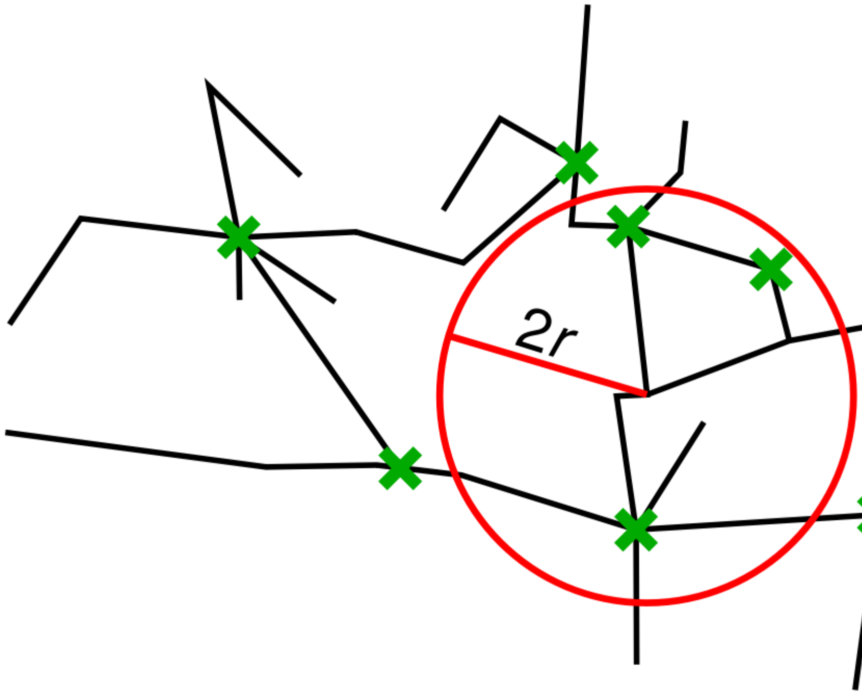

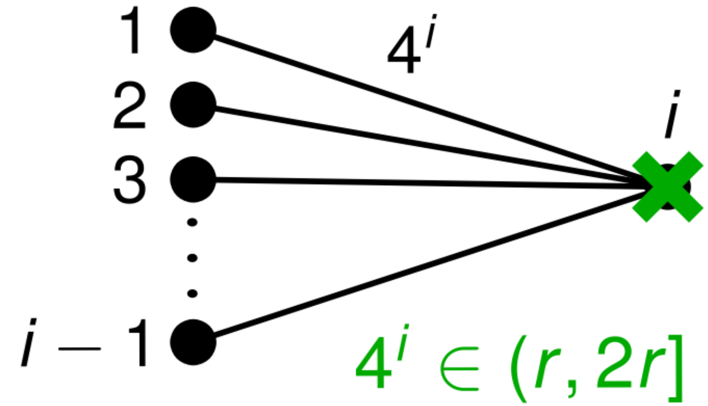

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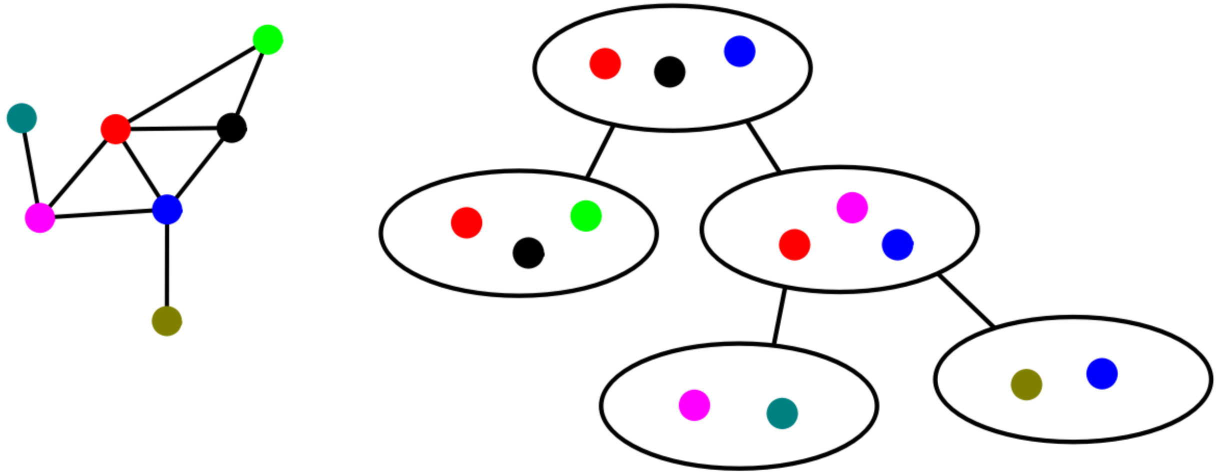

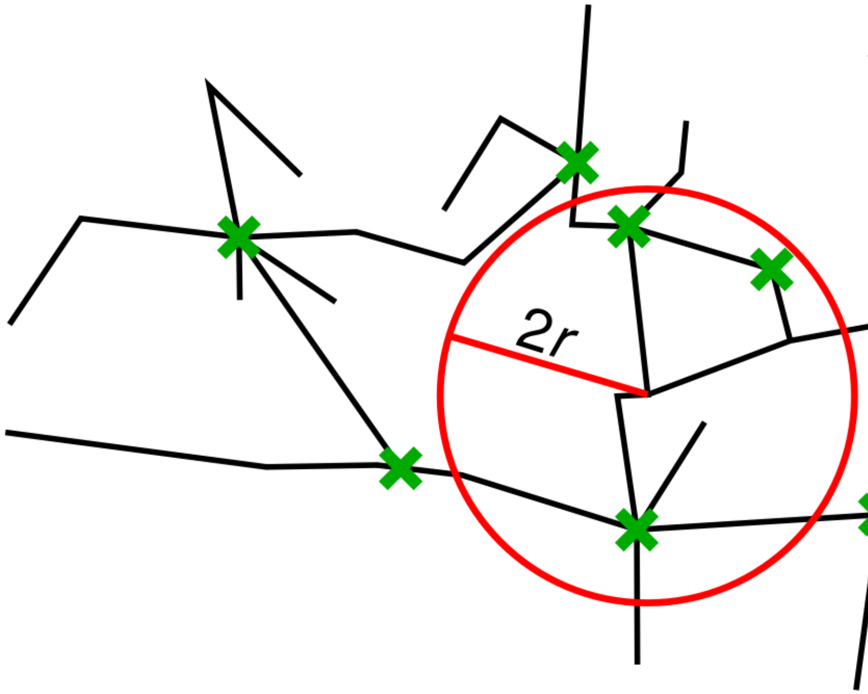

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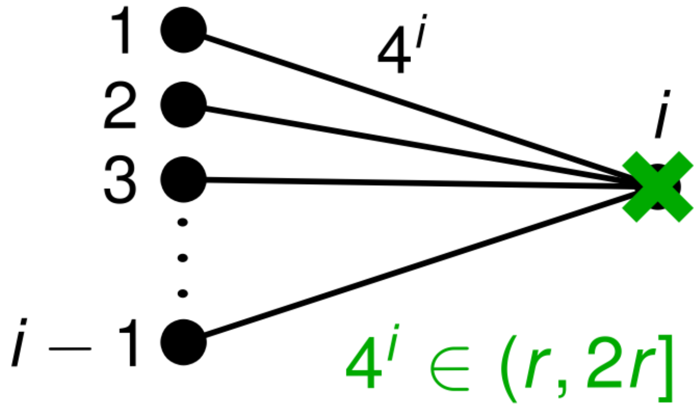
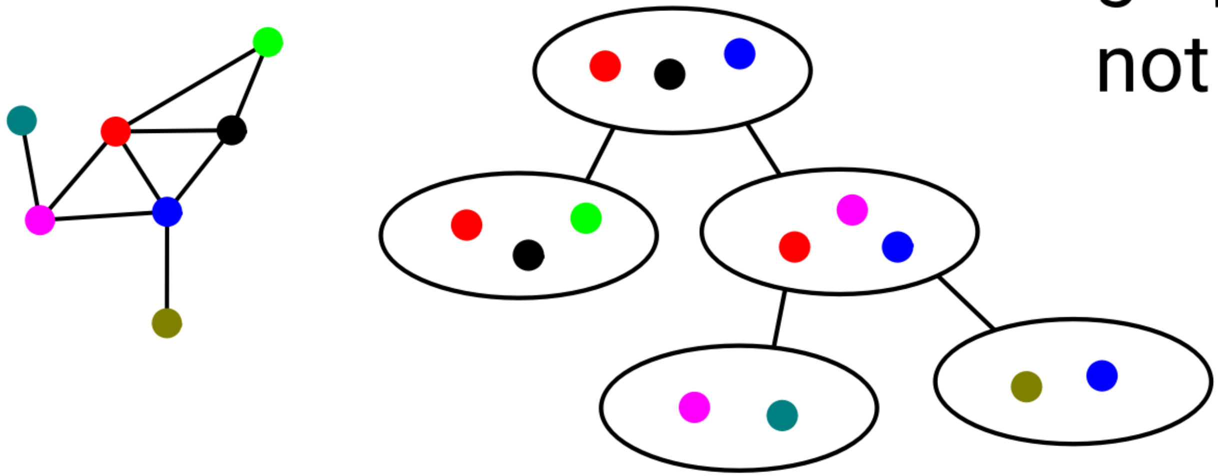


Treewidth $\not\Leftarrow$

graphs of hd=1 are not minor-closed



Theorem

TSP is NP-hard on graphs of highway dimension 6.

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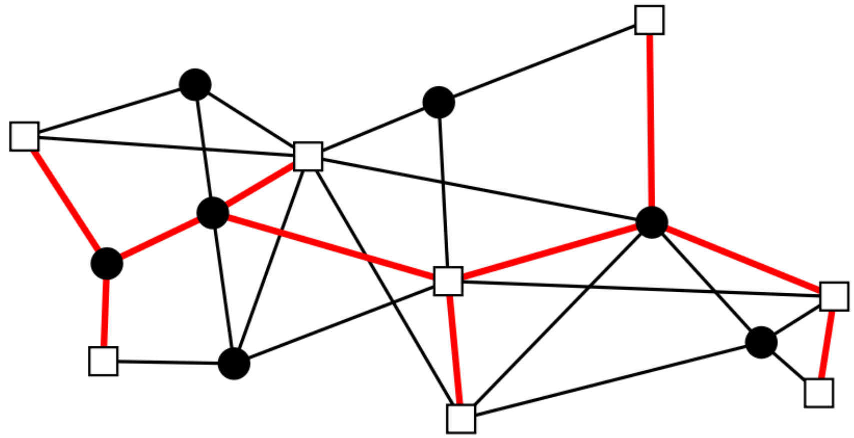
Steiner Tree:
– given:
- graph $G = (V, E)$
- edge weights $w : E \to \mathbb{R}^+$
- terminals $R \subseteq V$

– find:
- tree $T \subseteq G$ containing R
- min $c(T)$



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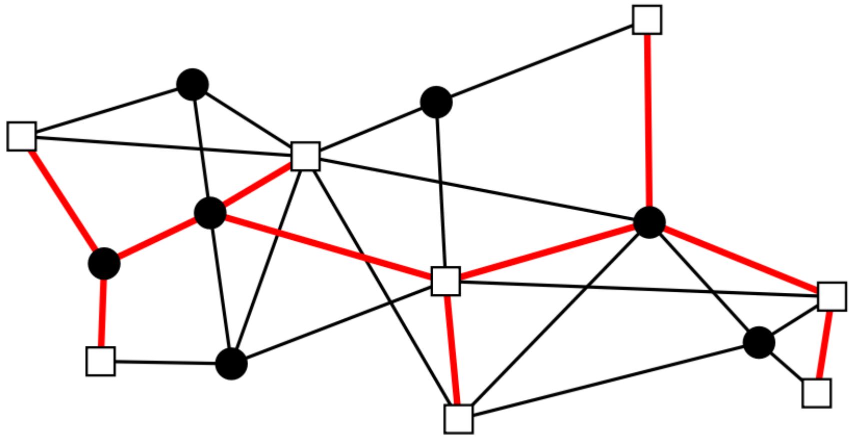
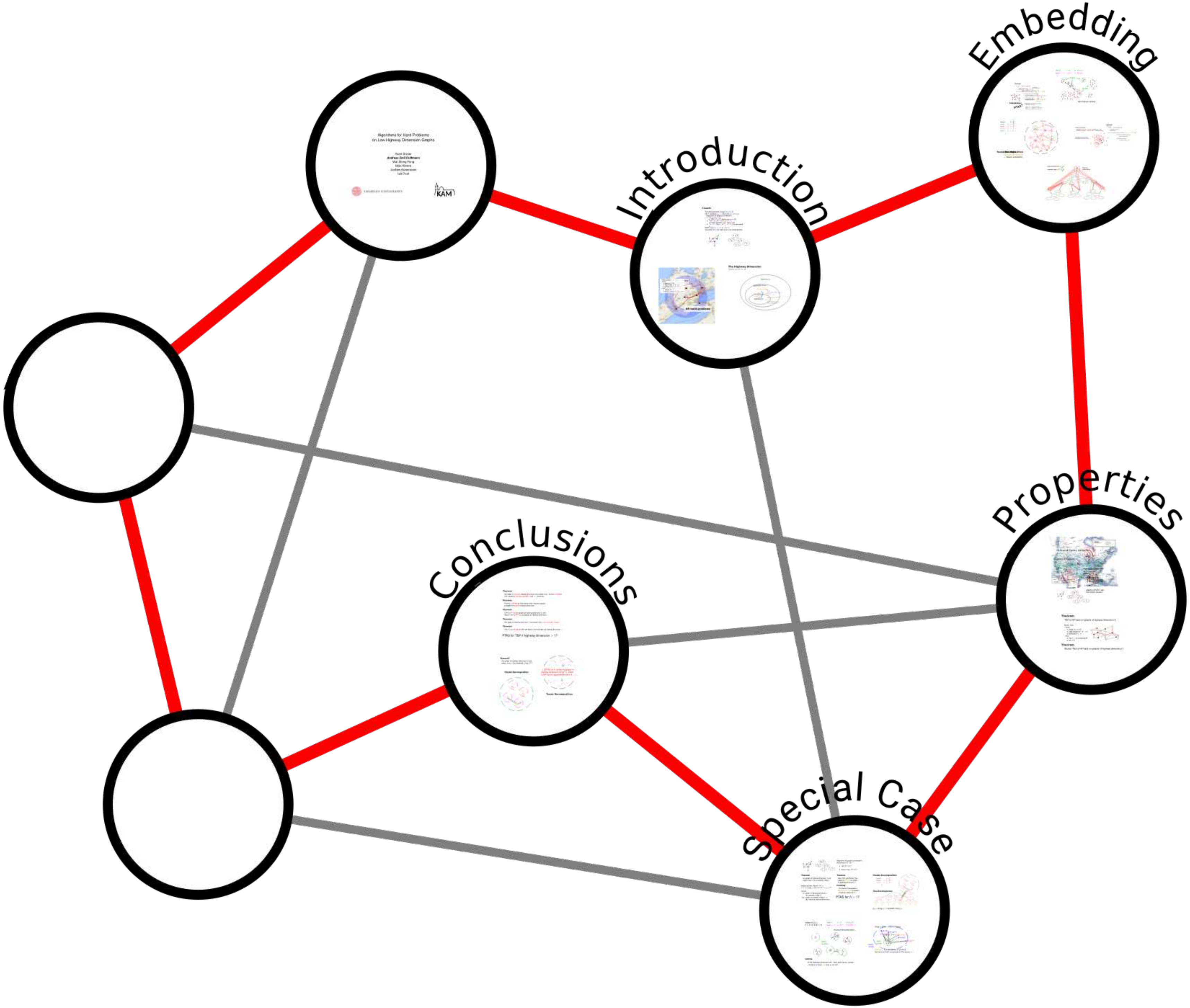
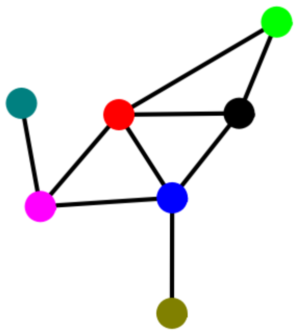
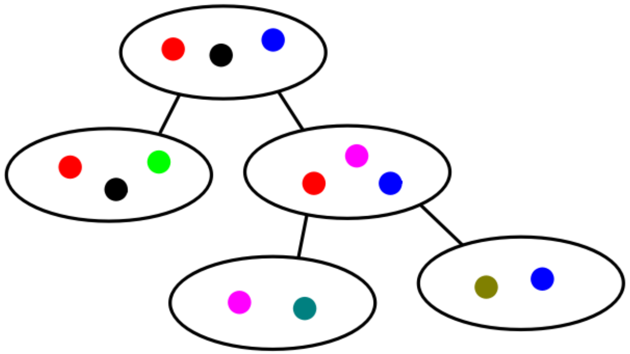


Theorem

Steiner Tree is NP-hard on graphs of highway dimension 1.

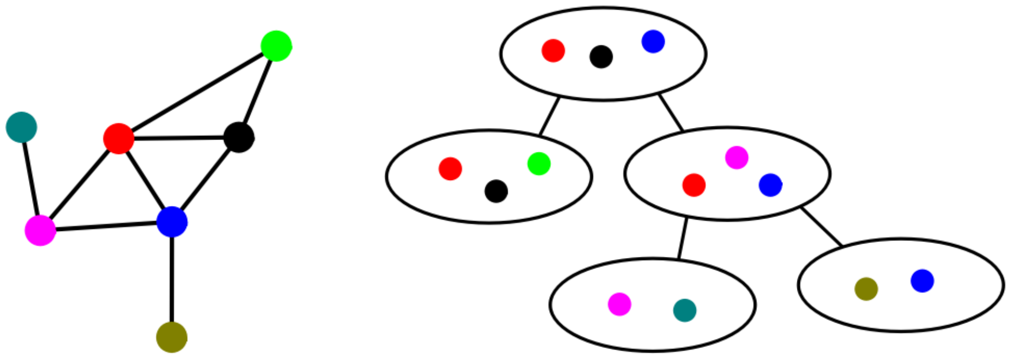

Algorithms for Hard Problems
on Low Highway Dimension Graphs
Introduction
Embedding
Properties
Conclusions
Special Case





Algorithms for graphs of treewidth t:
[Bodlaender et al. '03]

- TSP: $2^{O(t)} n^{O(1)}$
- Steiner Tree: $2^{O(t)} n^{O(1)}$



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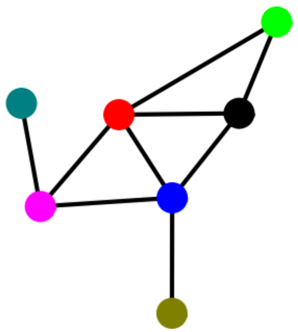

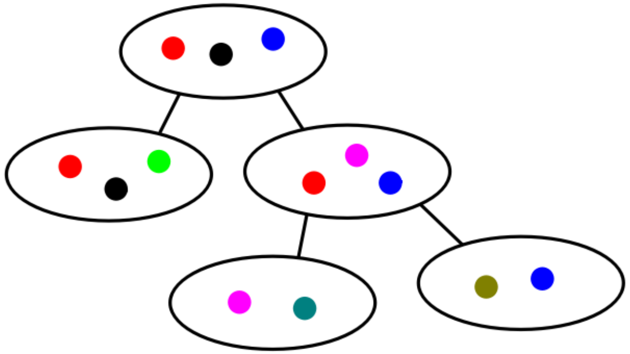

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preprocessing: reduce α to n/ε
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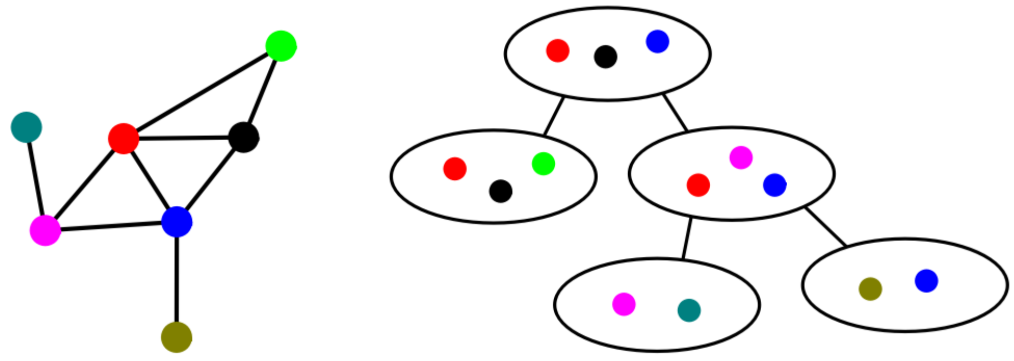

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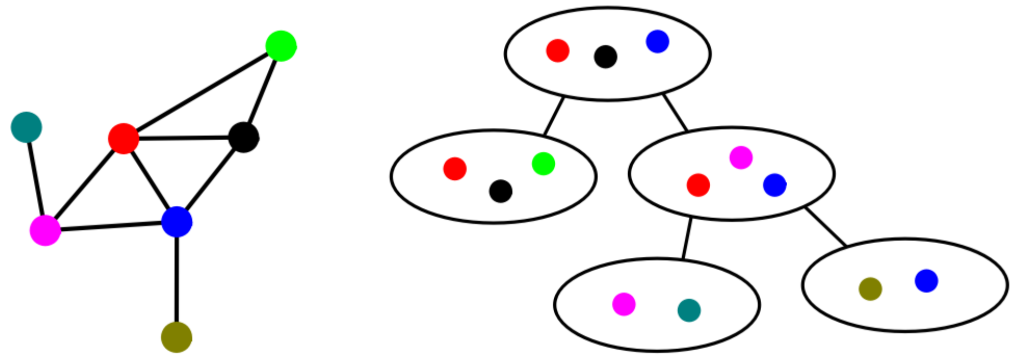

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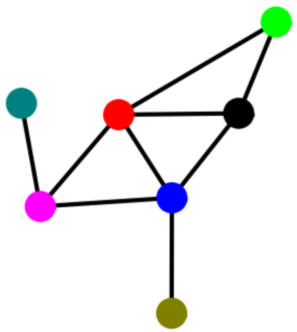
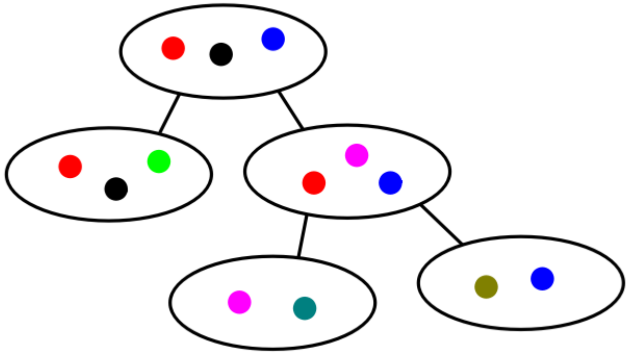
The Steiner Tree problem is weakly NP-hard on graphs of highway dimension 1.





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caveat:
in: graph of highway dimension 1 (of treewidth $O(\log \alpha)$)
out: graph of treewidth $O(\log(n/\varepsilon))$ (but arbitrary highway dimension)

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$e \in E$ s.t. $w(e) \leq 2r$

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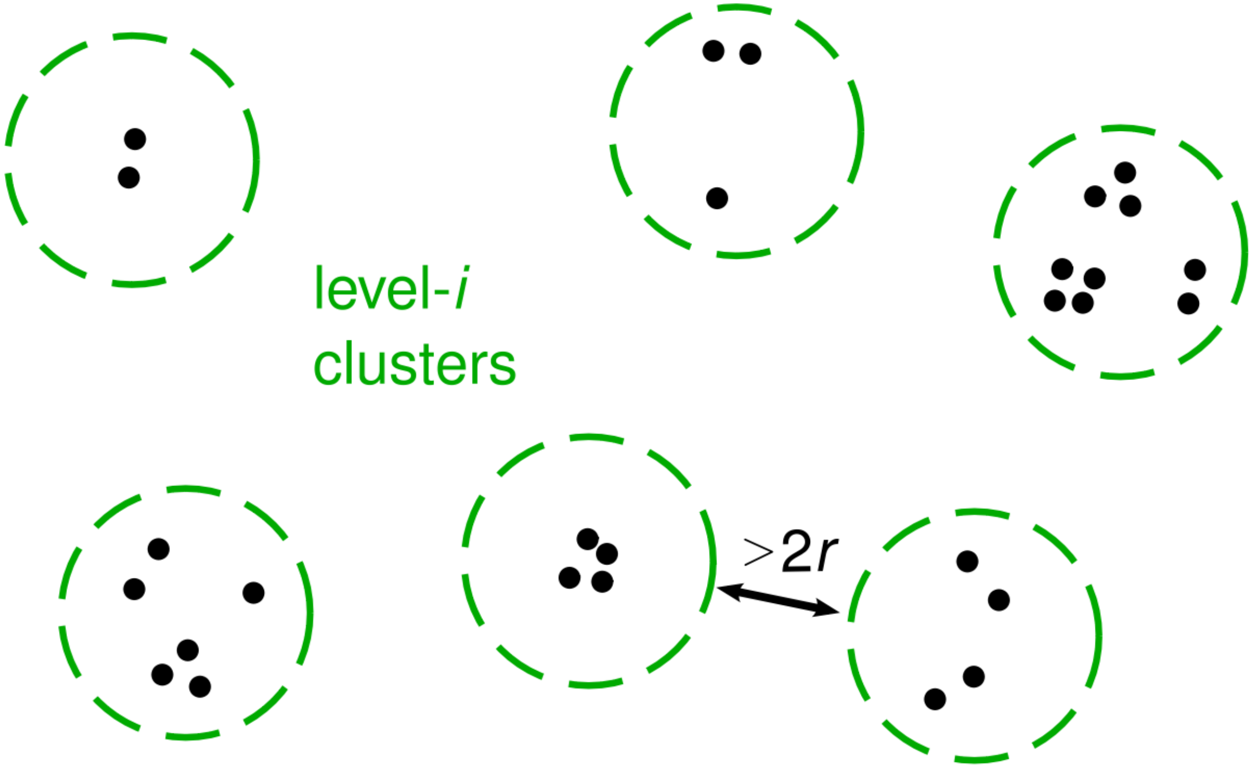

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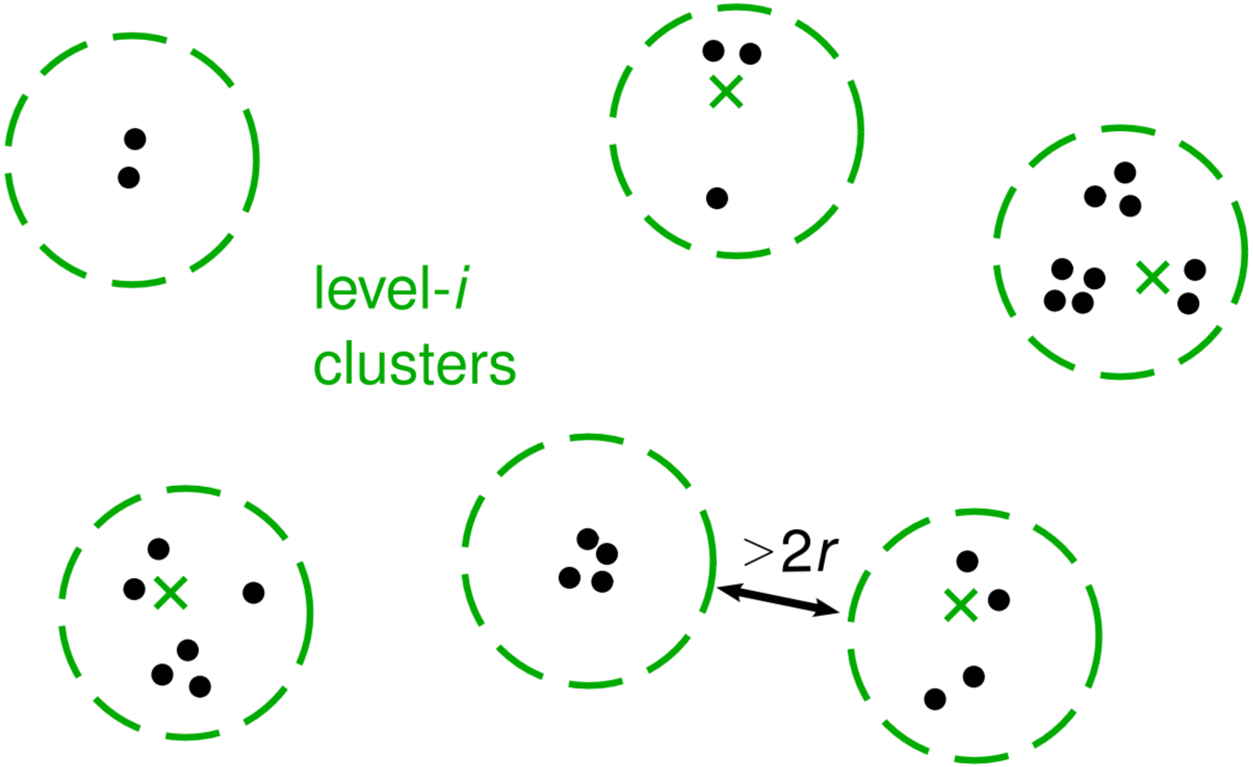
level i: $r = 2^i$ × $\textsc{spc}(2^i)$



Lemma

If the highway dimension is 1, then each level-i cluster contains at most one hub of $\textsc{spc}(2^i)$

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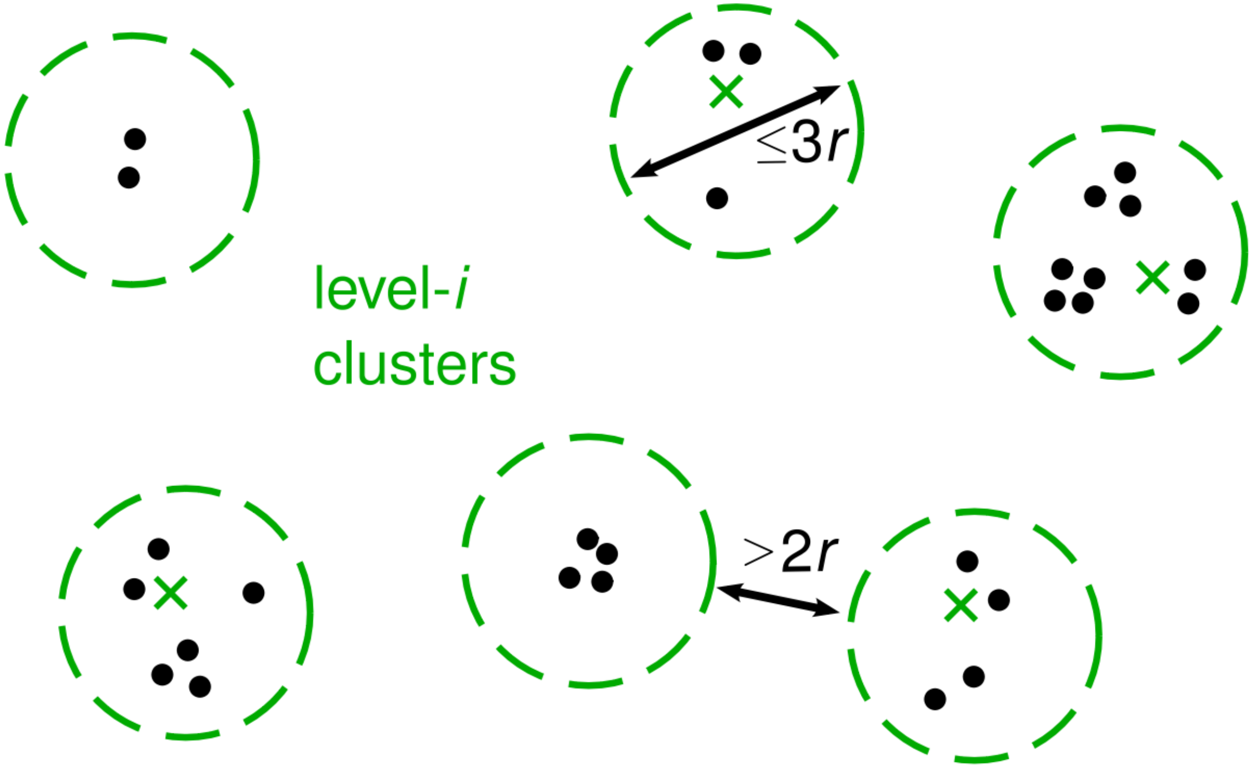

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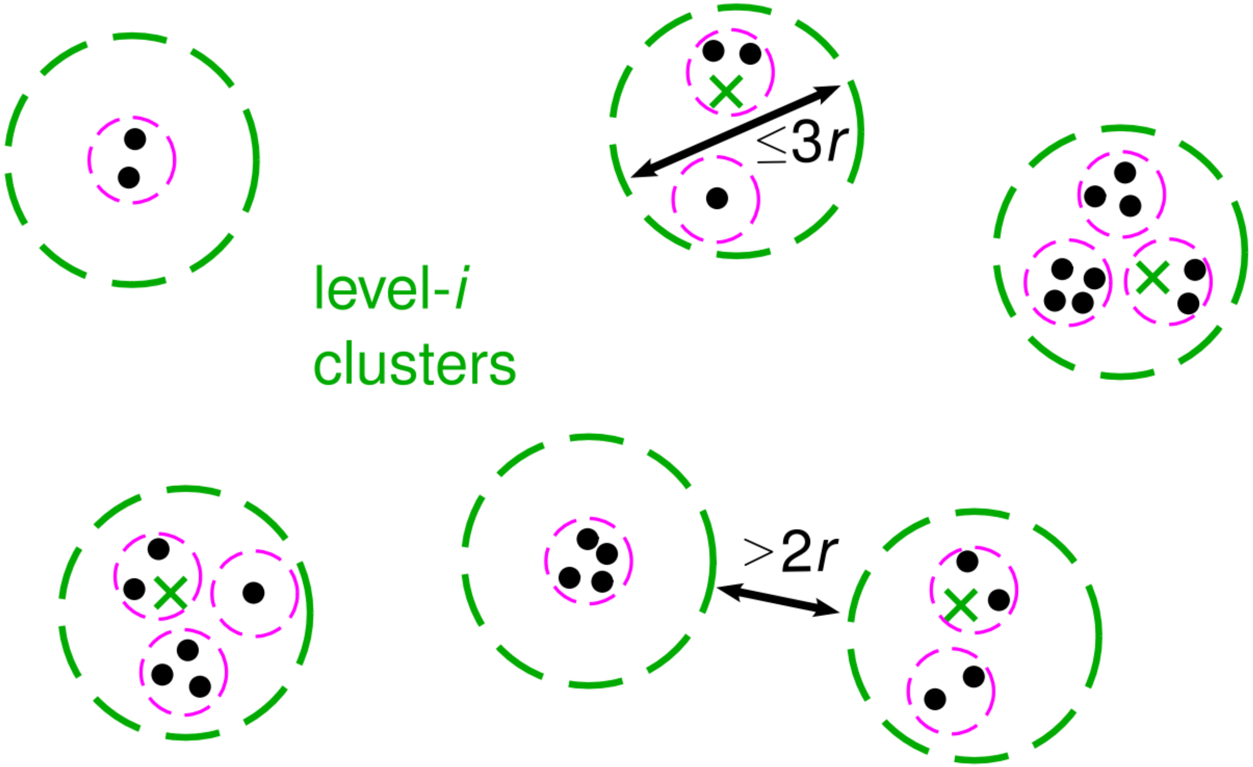

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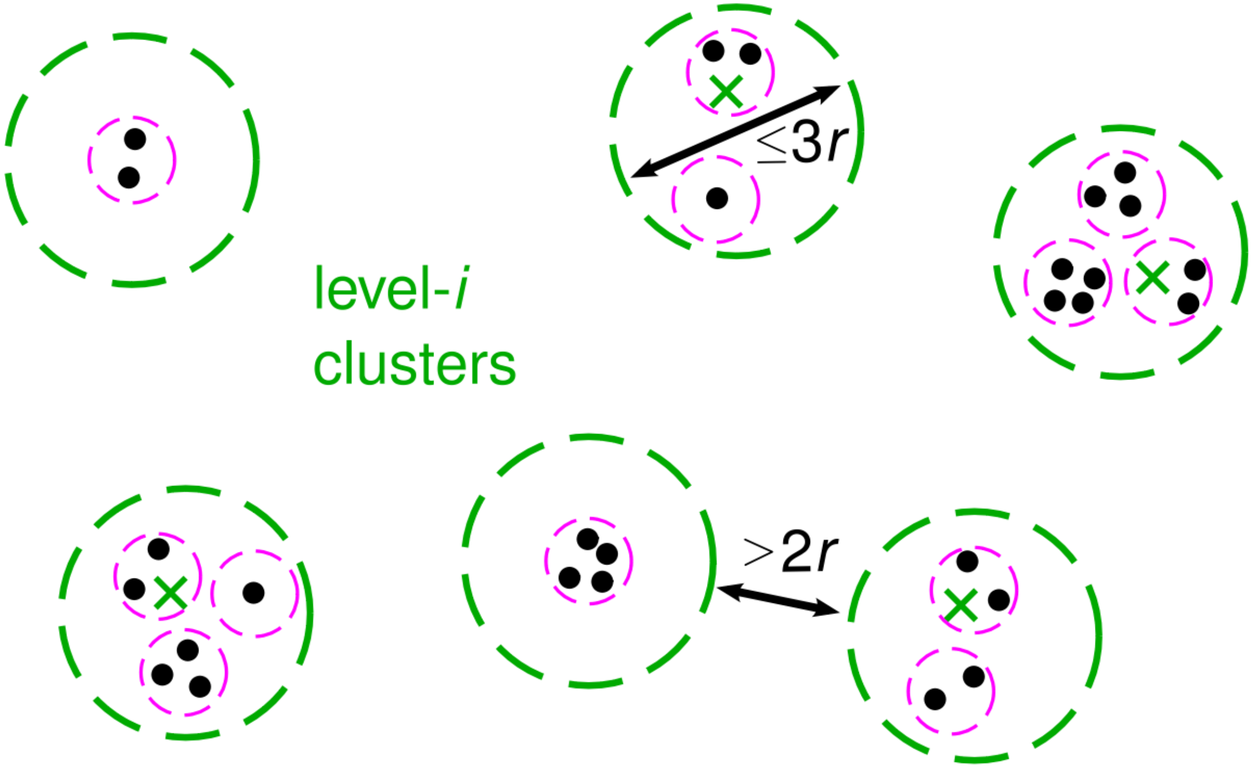
⋮

recursive decomposition...



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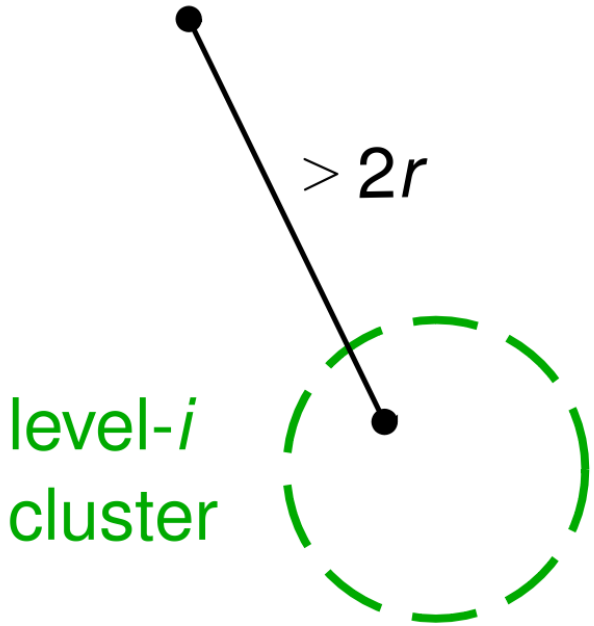
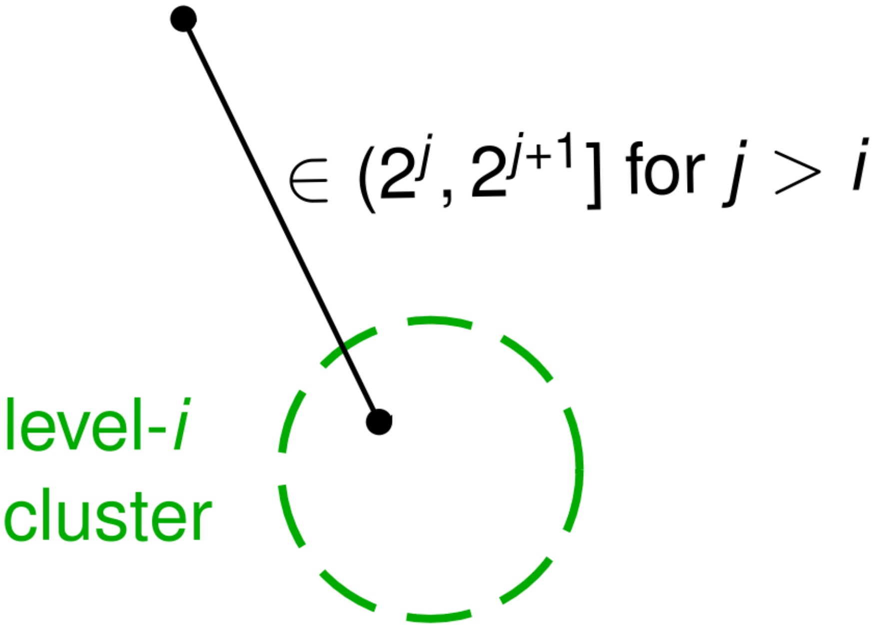
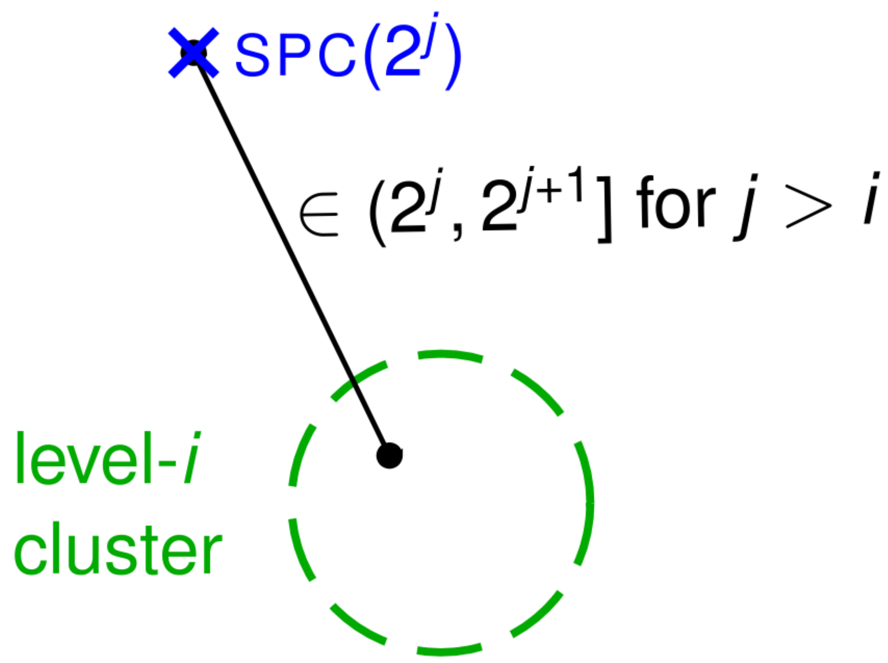

$> 2r$
level-i
cluster



$\in (2^j, 2^{j+1}]$ for $j > i$
level-i
cluster



SPC(2^j)
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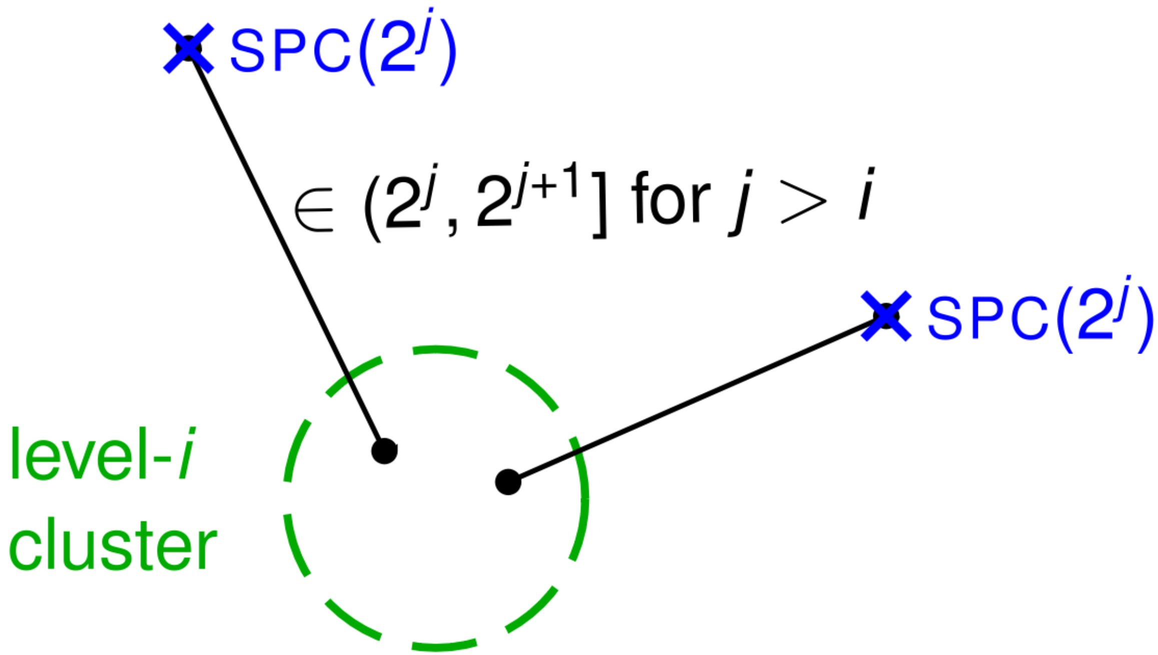
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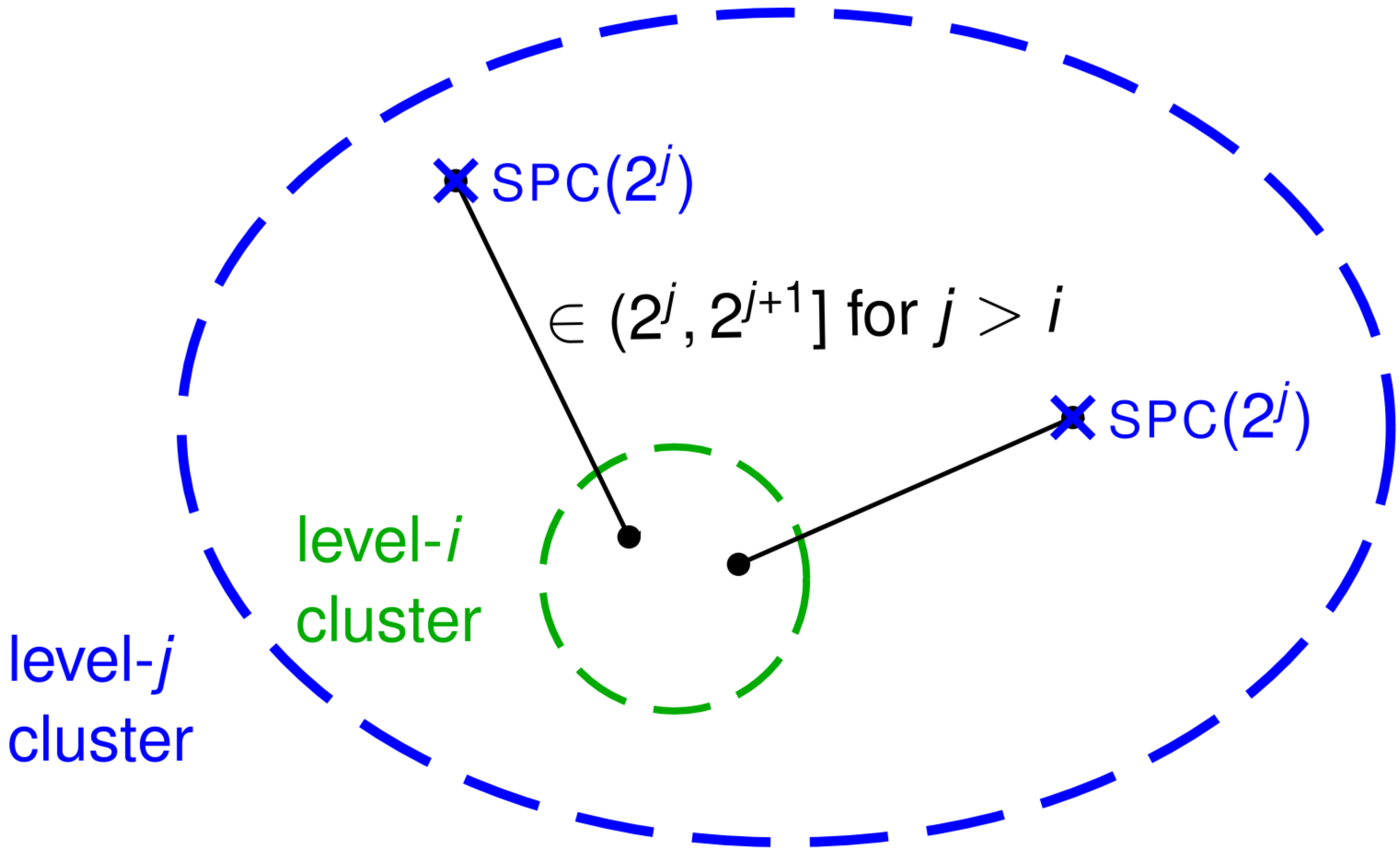

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cluster
level-j
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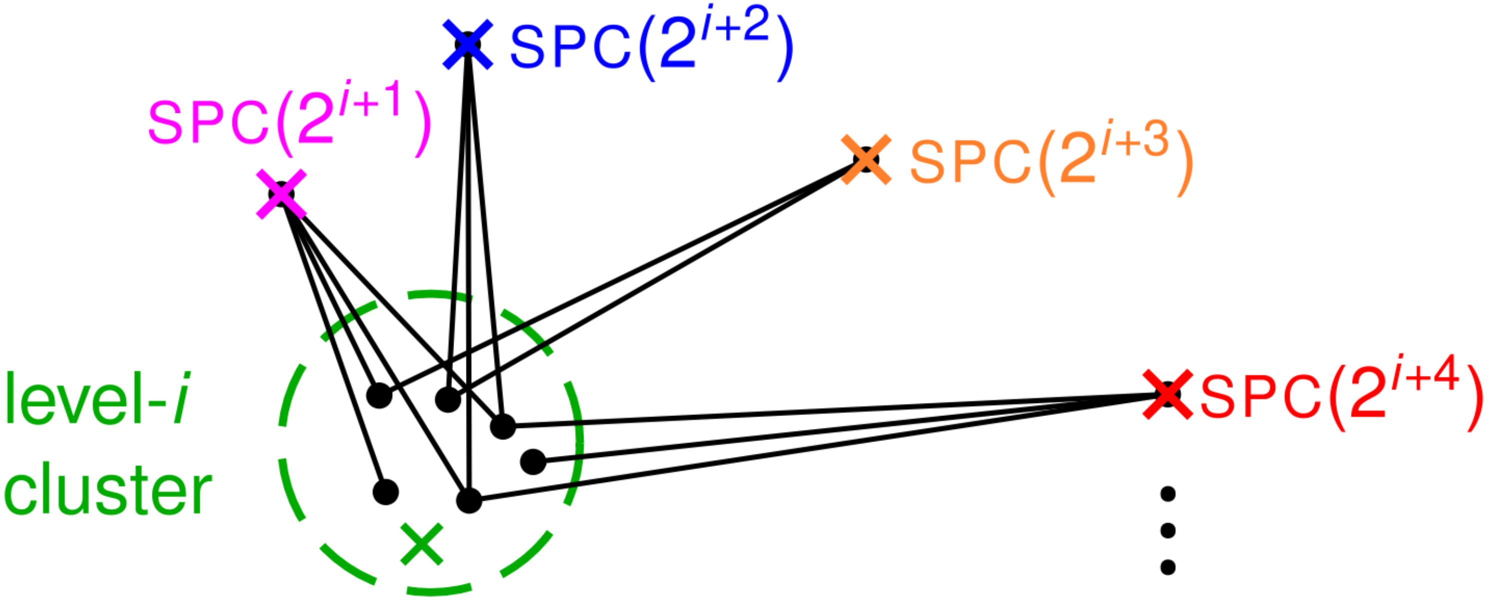
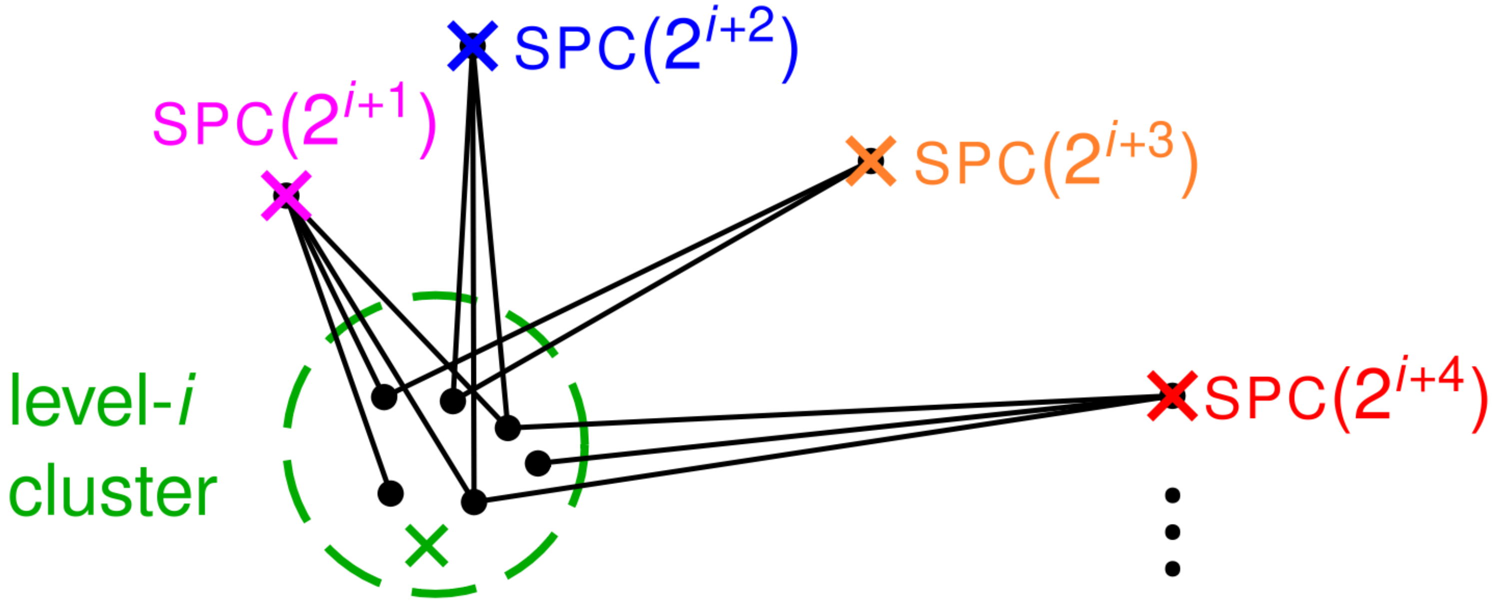


interface I_C of level-i cluster C contains
the hub of SPC(2^j) connected to C for every $j \geq i$

$O(\log \alpha)$ levels $\Rightarrow |I_C| = O(\log \alpha)$



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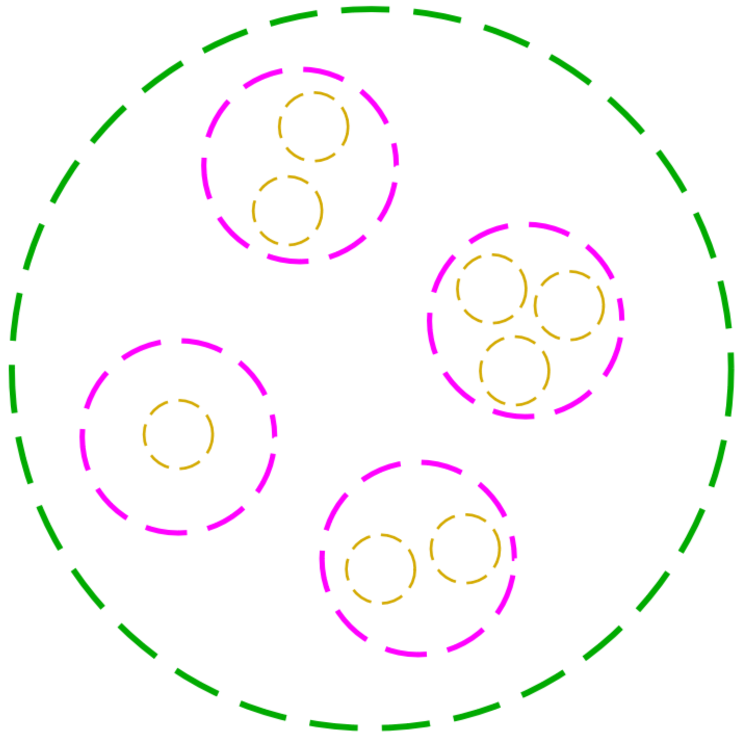
Cluster Decomposition

level i: $r = 2^i$

level $i-1$: $r = 2^{i-1}$

level $i-2$: $r = 2^{i-2}$

⋮



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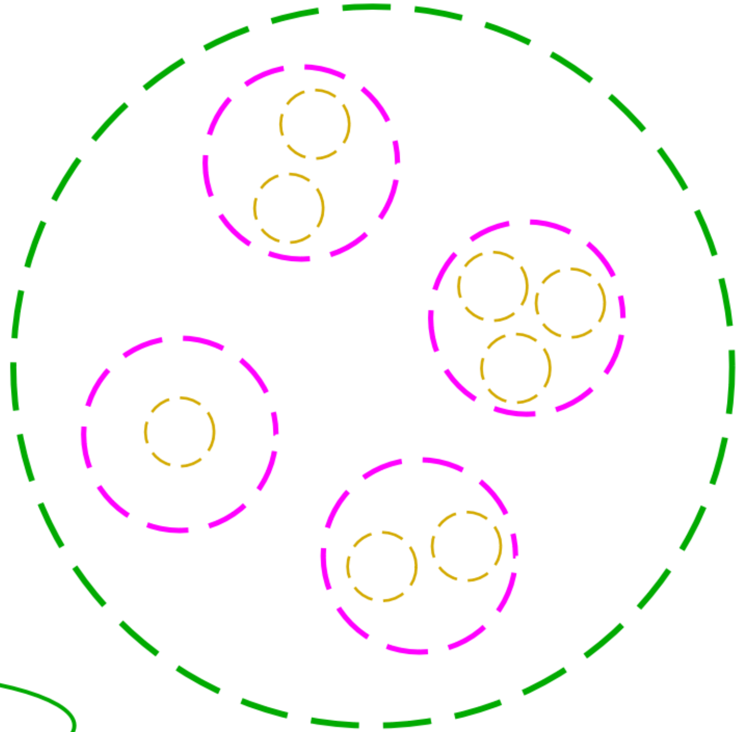
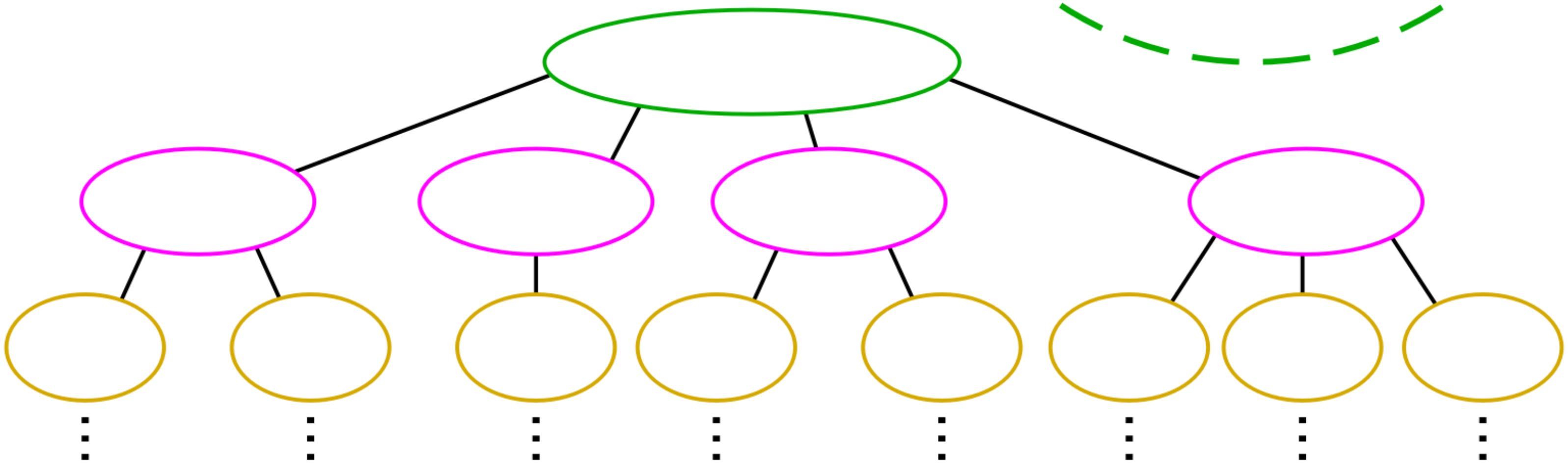
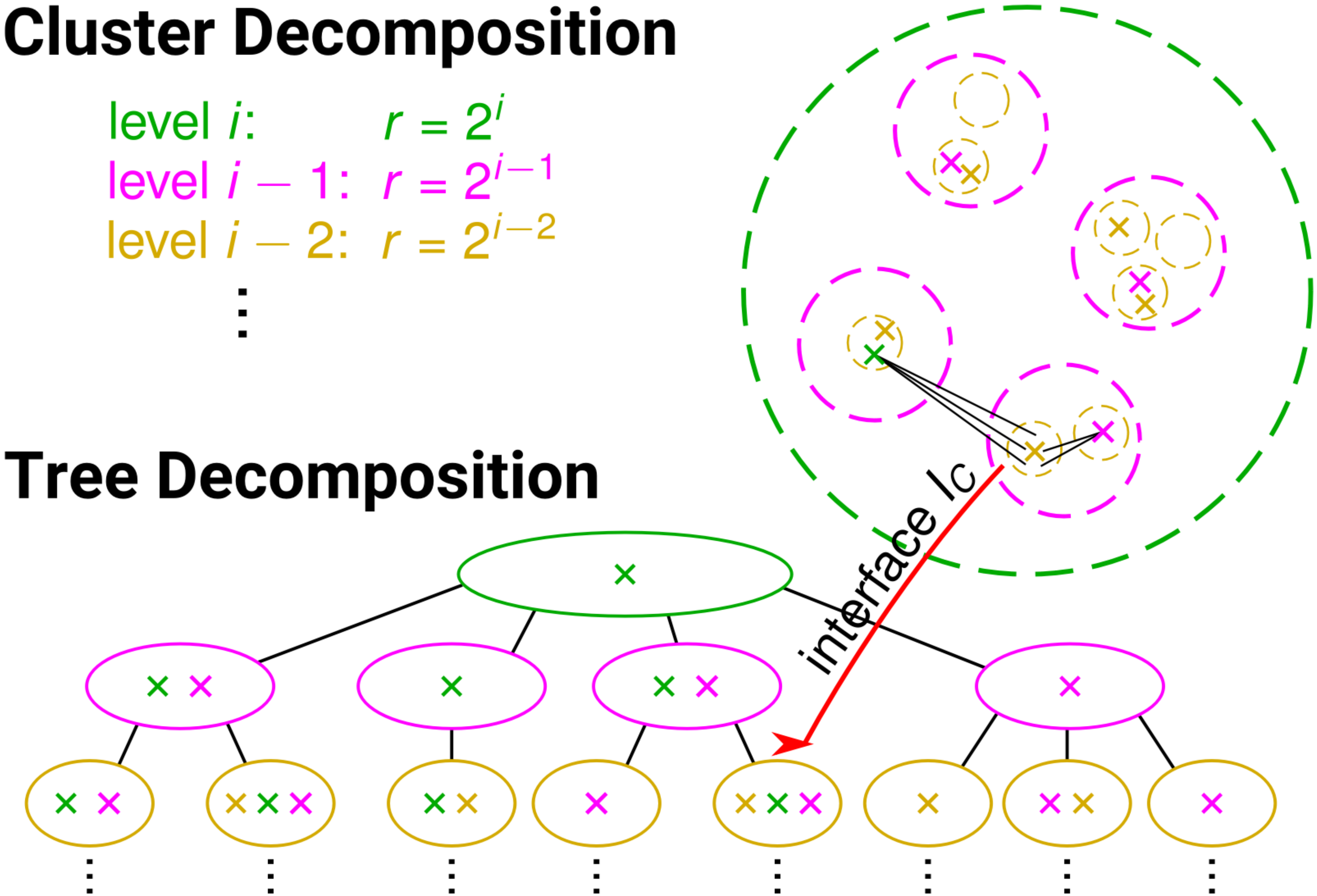


Tree Decomposition





Cluster Decomposition
level i: r = 2^i
level i − 1: r = 2^(i−1)
level i − 2: r = 2^(i−2)
Tree Decomposition
interface I_C

Cluster Decomposition

level i: $r = 2^i$

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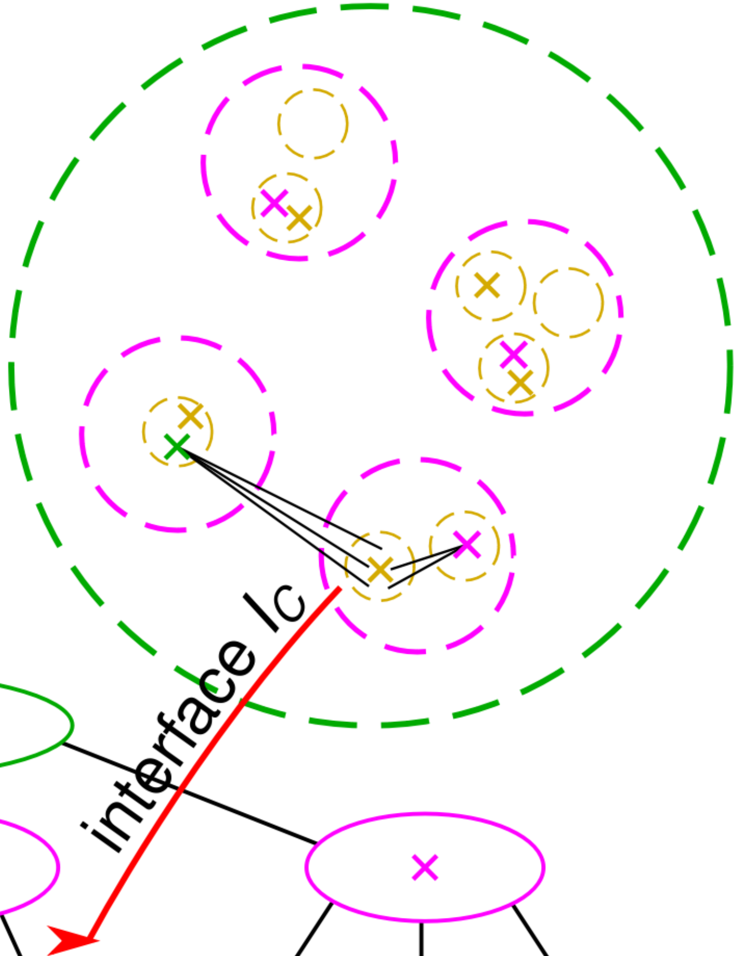
$\vdots$



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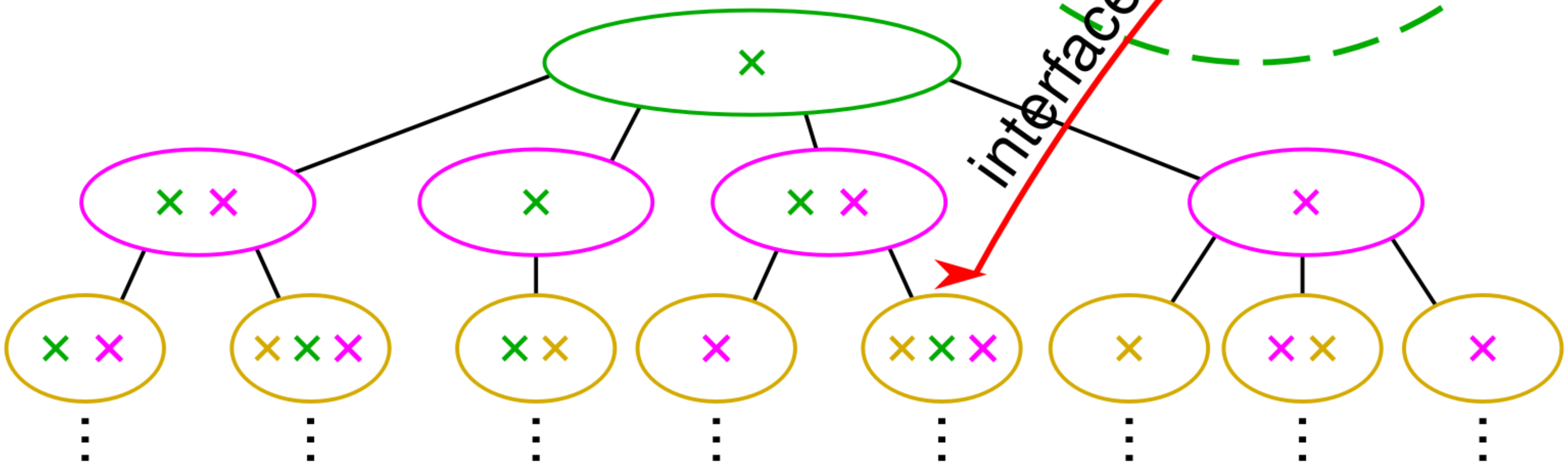
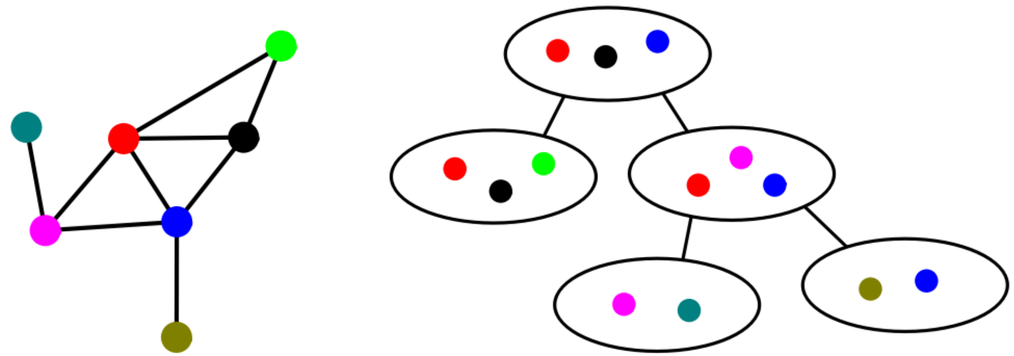


$|I_C| = O(\log \alpha) \Rightarrow$ treewidth $O(\log \alpha)$



Algorithms for graphs of treewidth t:
[Bodlaender et al. '03]

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Theorem

Any graph of highway dimension 1 and aspect ratio α has treewidth $O(\log \alpha)$.

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in: graph of highway dimension 1
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(but arbitrary highway dimension)

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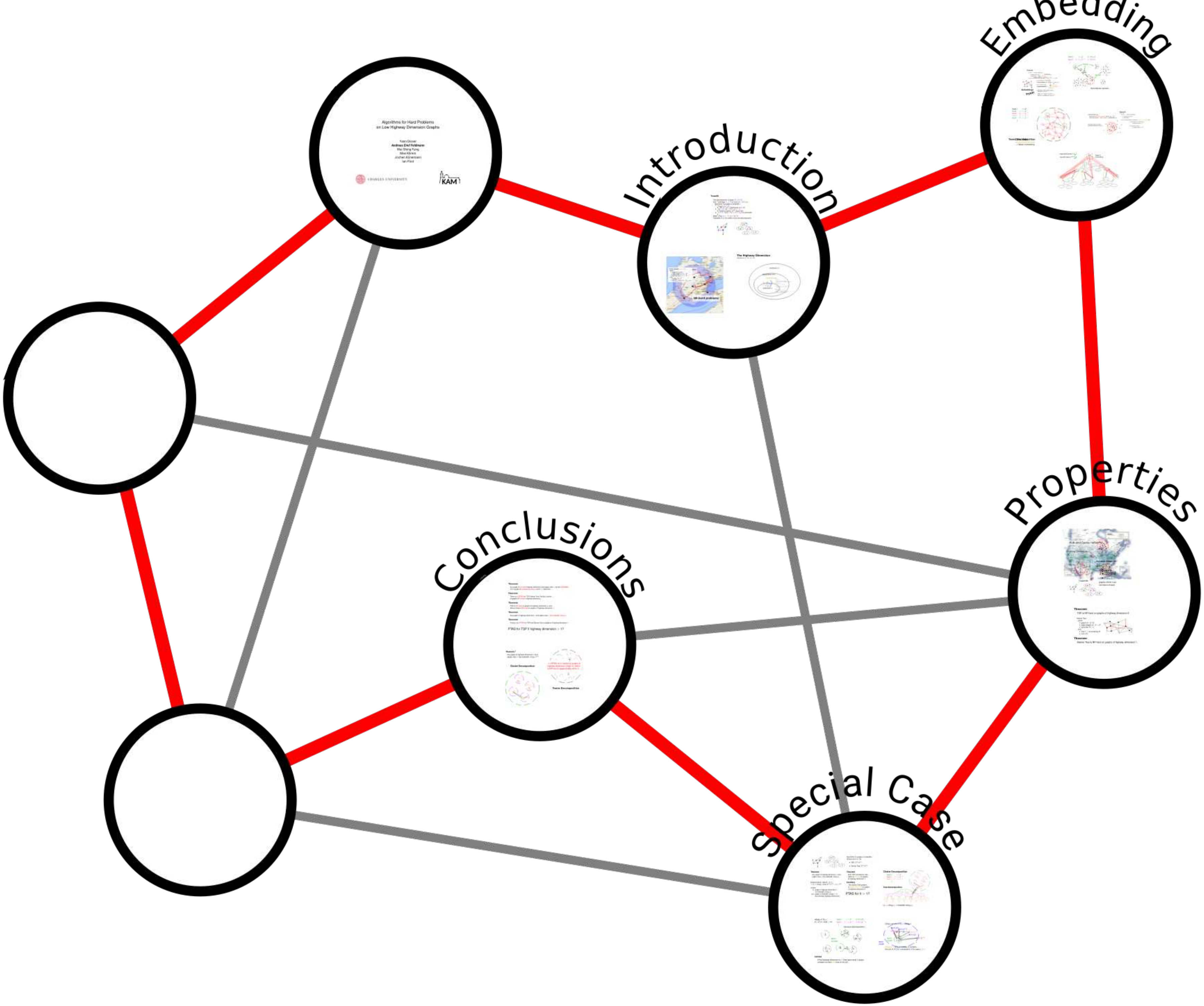
Corollary

The Steiner Tree problem is weakly NP-hard on graphs of highway dimension 1.

PTAS for $h > 1$?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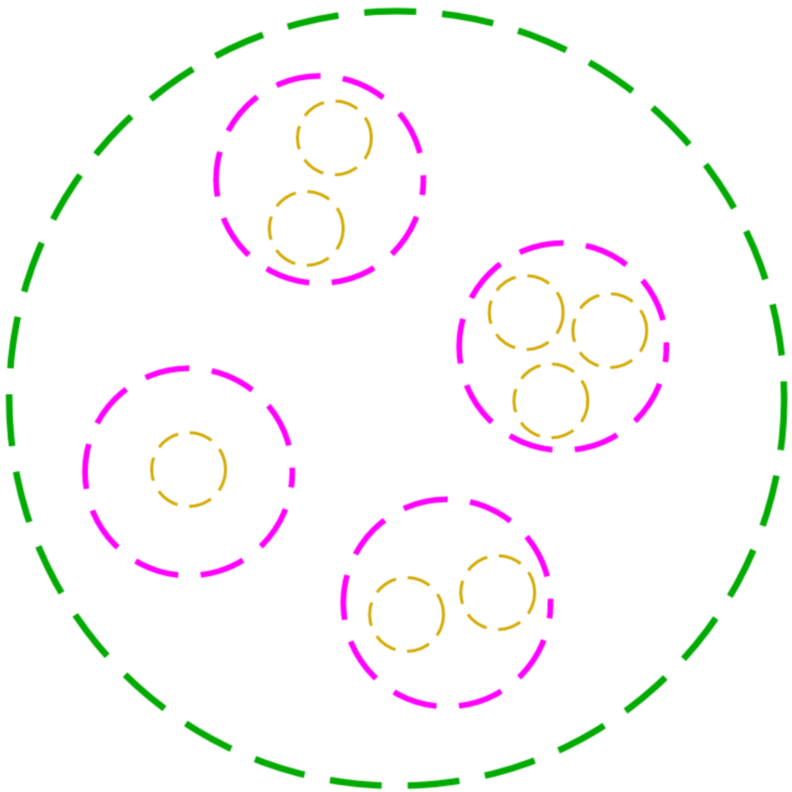
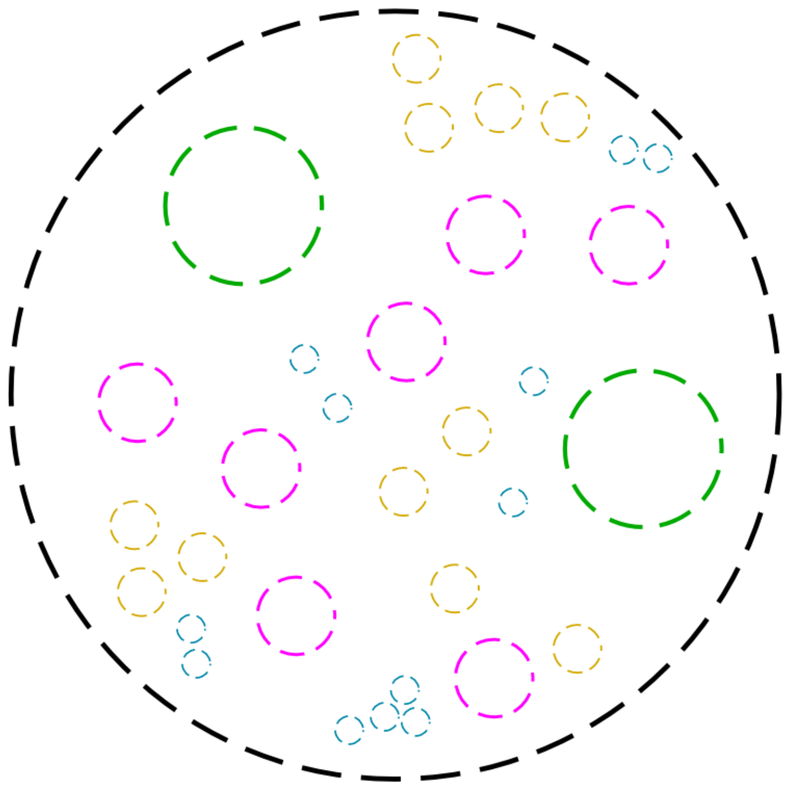
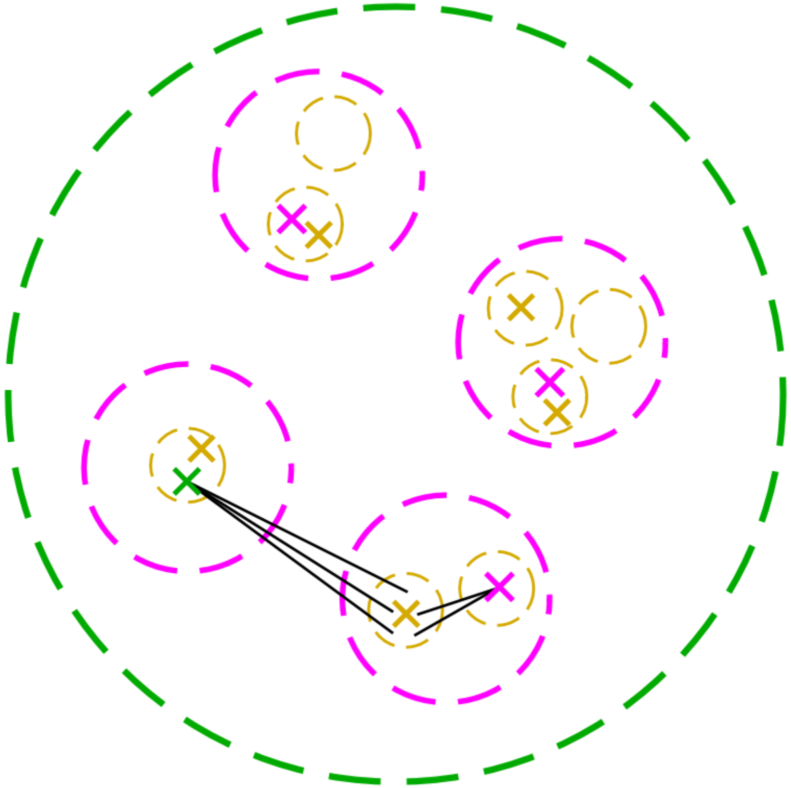




Towns Decomposition

Cluster Decomposition



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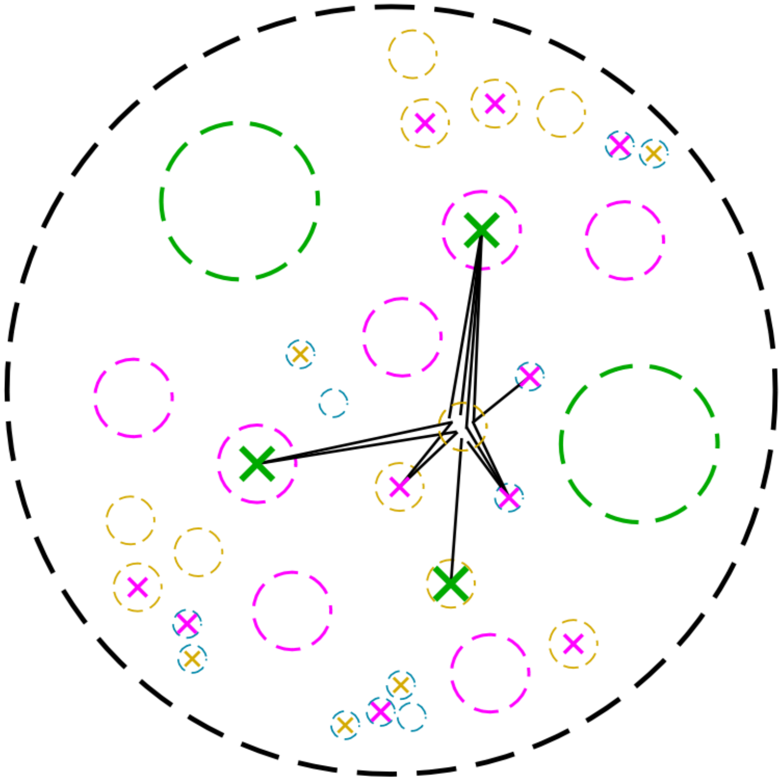
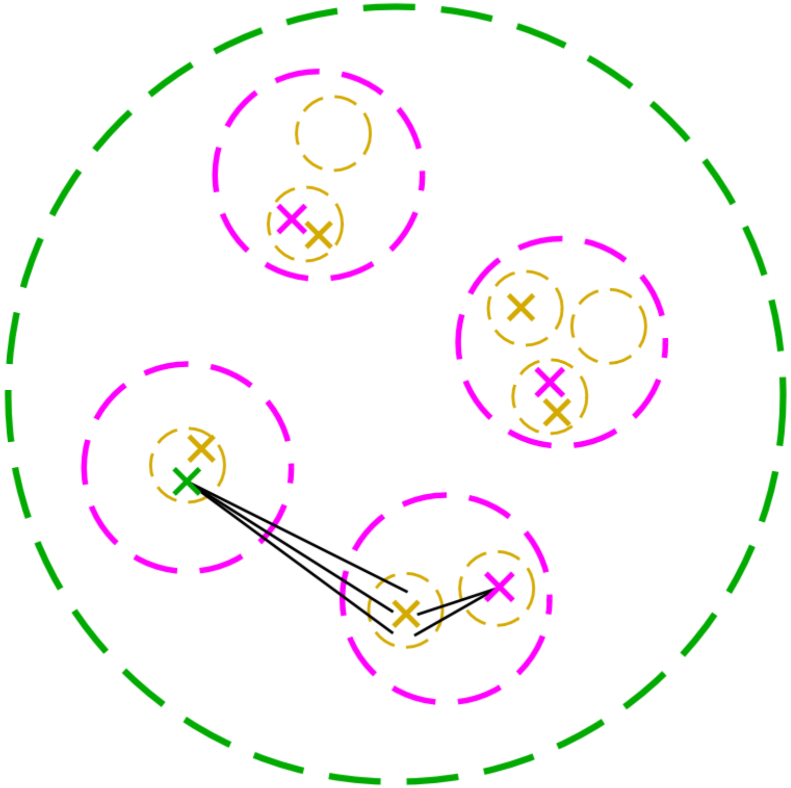
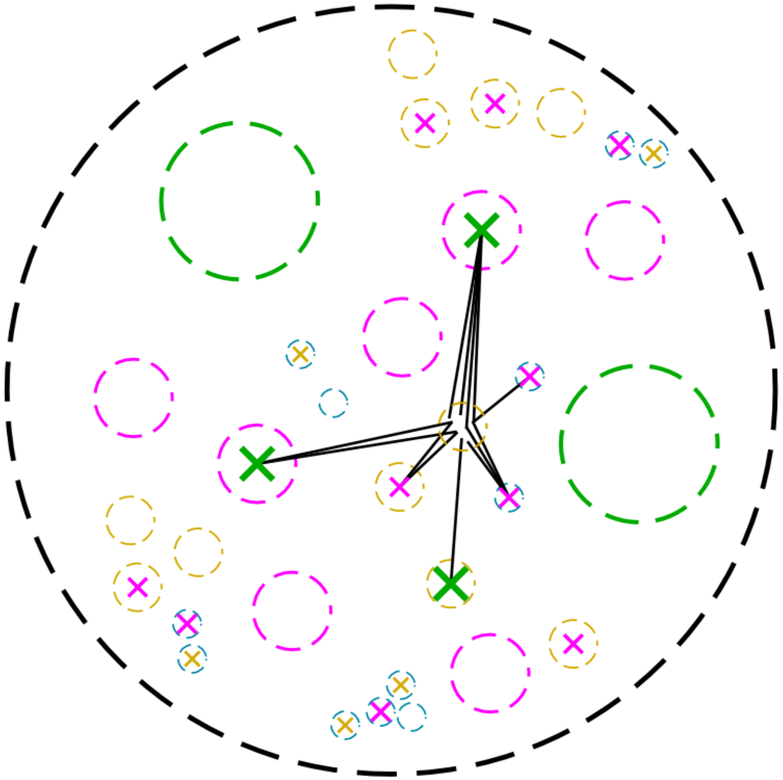


Theorem ?

Any graph of highway dimension h and aspect ratio α has treewidth $(h \log \alpha)^{O(1)}$.

Cluster Decomposition





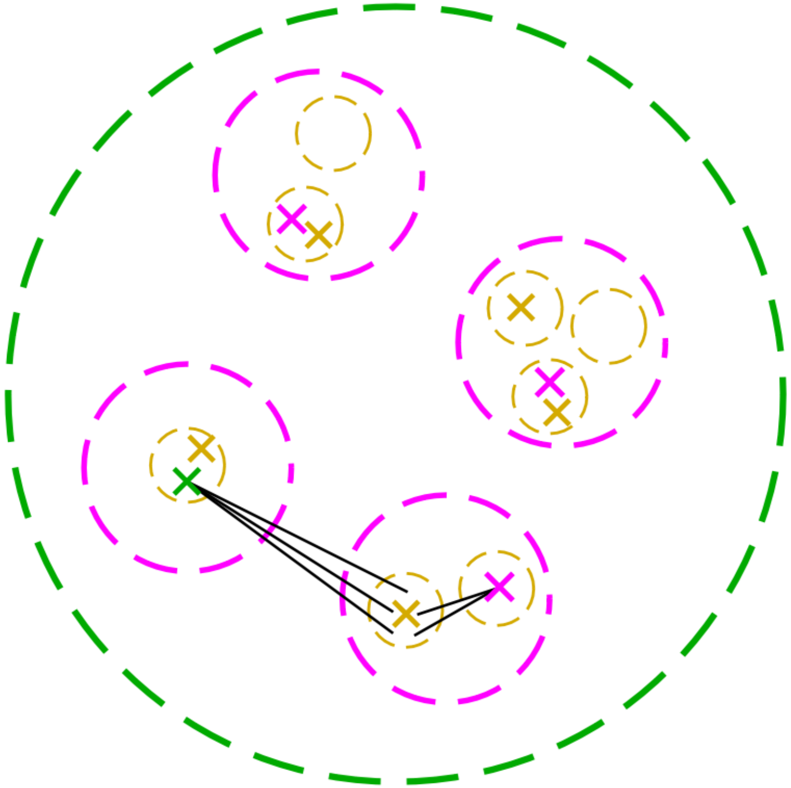
Towns Decomposition

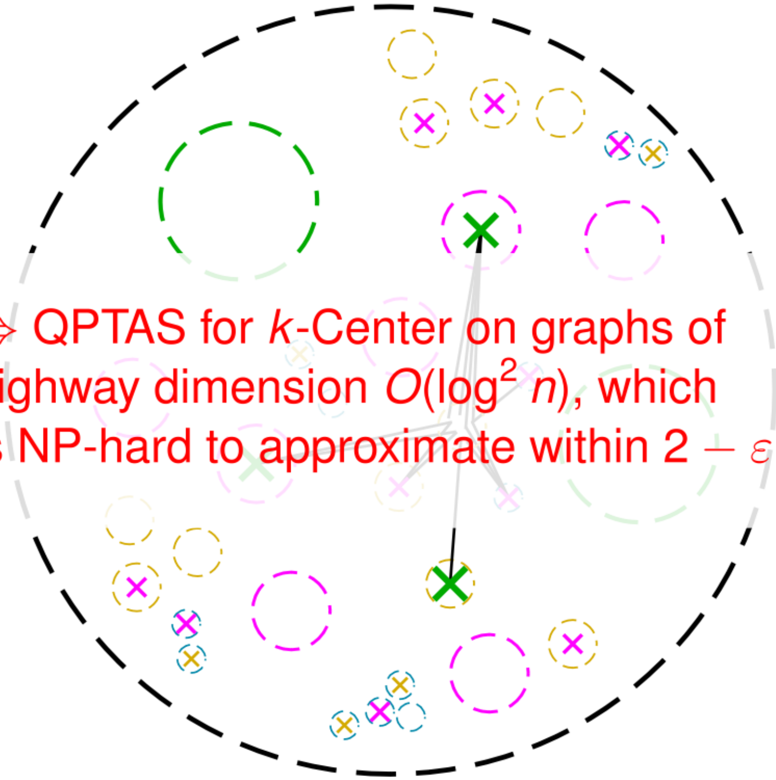
Theorem ?

Any graph of highway dimension h and aspect ratio α has treewidth $(h \log \alpha)^{O(1)}$.

$\Rightarrow$ QPTAS for k-Center on graphs of highway dimension $O(\log^2 n)$, which is NP-hard to approximate within $2 - \varepsilon$...

Cluster Decomposition





Towns Decomposition

Theorem

Any graph of constant highway dimension and aspect ratio α can be embedded into a graph of treewidth polylog(α) and $1 + \varepsilon$ distortion.

Theorem

There is a QPTAS for TSP, Steiner Tree, Facility Location, ...
on graphs of constant highway dimension.

Theorem

TSP is NP-hard on graphs of highway dimension 6, and
Steiner Tree is NP-hard on graphs of highway dimension 1.

Theorem

Any graph of highway dimension 1 and aspect ratio α has treewidth $O(\log \alpha)$.

Theorem

There is an FPTAS for TSP and Steiner Tree on graphs of highway dimension 1.

PTAS for TSP if highway dimension > 1?

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Thanks!